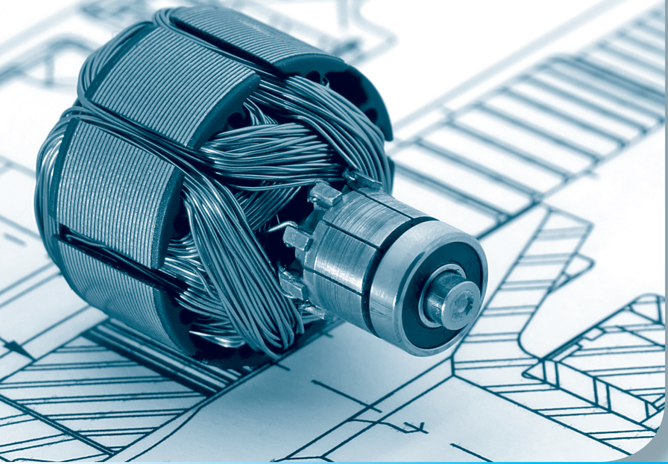

3

Three-Phase Induction Machines

What You Will Learn in This Chapter

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3. Rotating magnetic fields
4. Mechanical and electrical radians
5. Equivalent circuit
6. Torque and power relationships
7. Industrial considerations
8. Mechanical loads
9. Voltage, efficiency, and power factor
10. Actual motor characteristics
11. Measurement of equivalent circuit parameters
12. Special applications
13. Asynchronous generators
14. Electromechanical transients
15. Plugging
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2. Equivalent circuit
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3.0 Introduction

Three-phase induction motors are by far the most widely used motors in industry, accounting for about 80% of the total number of motors used in the average plant.* In many small and medium-sized industries, all motors above 3 kW are three-phase induction motors. Three-phase induction motors are popular because they are more economical, last longer, and require less maintenance than other types of motors. Because of their importance, this chapter provides a detailed analysis of three-phase motors and a thorough discussion of their applications. The following subjects are discussed: principles of operation, rotating magnetic fields, equivalent circuits, governing mathematical relationships, industrial considerations, induction generators, and solid-state control.

Manufacturers' data for three-phase induction machines are included in the tables in Section 3.7 at the end of this chapter. Their control schematics are extensively covered in Chapter 7. The physical distribution of the machine windings and their unbalanced voltage operation are covered in more specialized textbooks.

3.1 Three-Phase Induction Motors

3.1.1 Stator and Rotor

Like all motors, three-phase induction motors transform electrical power into mechanical power by means of a stationary part called the stator and a rotating part called the rotor. Stator or *armature windings* are housed on the stator, whereas *rotor windings* are installed on the rotor. For an elementary representation of stator and rotor windings, see Fig. 3-1.

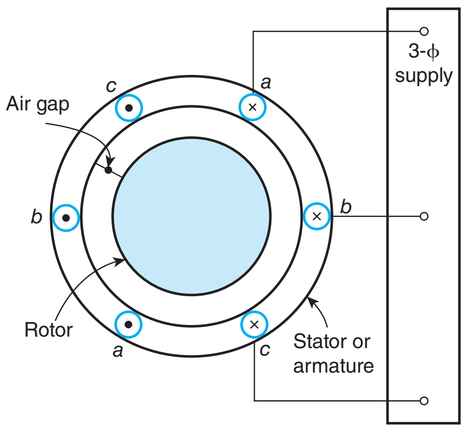

FIG. 3-1 An elementary representation of a three-phase induction motor. (In an actual machine, the armature and the field windings are placed in the stator and rotor slots, respectively.)

*Only about 50% of the energy used by a plant is consumed by 3-ϕ induction motors. Roughly 35% is absorbed by synchronous and dc machines, and the remaining 15% is consumed by single-phase motors, heating,cooling, lighting, and miscellaneous.

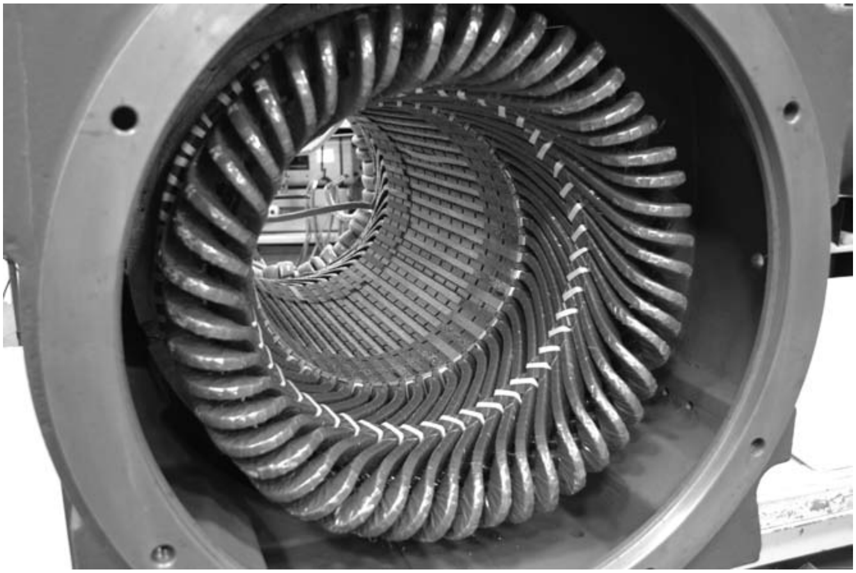

FIG. 3-2 Stator windings. *Photo courtesy of Siemens Industry, Inc.*

Stator windings can be either star- or delta-connected. Their main functions are to receive three-phase ac power and to produce a *single rotating* magnetic field that has an approximately sinusoidal space distribution. This rotating field completes its magnetic path through the stator, two air gaps, and the rotor structure. Figure 3-2 shows a manufacturer's picture of an actual armature winding. The stator windings are brought out in a terminal box where they can be connected to a suitable three-phase voltage supply.

There are two types of rotor windings: the squirrel cage and the wound type. Manufacturer's pictures of squirrel-cage and wound-rotor windings are shown in Figs. 3-3 and 3-4, respectively.

A *squirrel-cage* rotor winding is generally made up of bare aluminum bars that are connected at their terminals to shorted end rings. In other words, the rotor windings are always short-circuited regardless of motor operating condition. The rotor bars are not parallel to the rotor axis but are set at a slight skew. This feature reduces mechanical vibrations, making the motor less noisy.

Rotor windings cannot be electrically connected to a circuit outside the rotor. As a result, the rotor resistance is constant, and for a given stator voltage, its torque-speed characteristic is fixed (see Section 3.1.6).

In some special designs (for a particular motor application, for instance), the rotor may have double squirrel-cage windings, each with a different resistance. This construction gives higher starting torque, lower starting current, and higher full-load power factor.

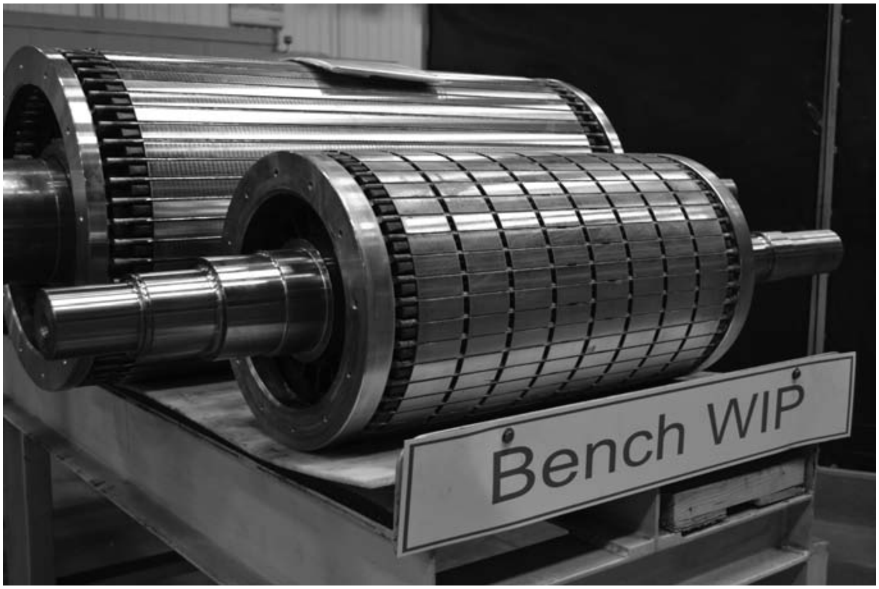

FIG. 3-3 Squirrel-cage rotor. *Photo courtesy of Siemens Industry, Inc.*

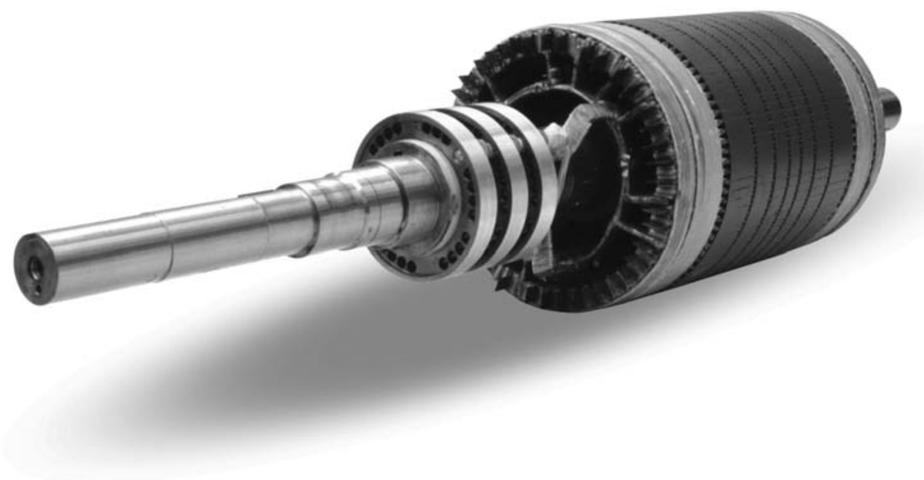

FIG. 3-4 Wound-rotor winding. *Courtesy of General Electric*

FIG. 3-5 Three-phase induction motors: 200 kW, 460 V. *Courtesy of General Electric*

FIG. 3-6 Induction motor rotors: 6300 kW, 4160 V. *Courtesy of General Electric*

A very small percentage of induction machines have a wound rotor. *Wound-rotor windings* terminate at the slip rings on which the brushes rest. The brushes can then be connected to a three-phase variable resistor, and the resistance of the rotor winding can be externally controlled. This variable resistor, as demonstrated in this chapter, controls the torque-speed characteristic of the motor. Figures 3-5 and 3-6 show, respectively, manufacturer's pictures of three-phase induction motors and rotors.

3.1.2 Principles of Operation

The operation of three-phase induction machines is based on the generation of a revolving field, the transformer action, and the alignment of the magnetic field axes.

When balanced three-phase currents are injected into the stator windings, they produce a *rotating magnetic field* that, unless the rotor is revolving at the same speed as the magnetic field, will *induce* voltages* in the rotor windings. This results in rotor current and therefore rotor flux. The magnetic fields of the stator and rotor try to *align their magnetic axes*—a natural phenomenon—and in so doing, a torque is developed.

Refer to Fig. 3-7. The flux of the synchronously rotating stator field (ϕ_s) and the flux of the rotor currents (ϕ_f) combine vectorially to produce the net or resultant flux (ϕ_R) within the structure of the stator and rotor.

From Ampere's Law (Eq. (1.161)), the force and thus the torque (T) developed by two interacting fields are proportional to the product of the

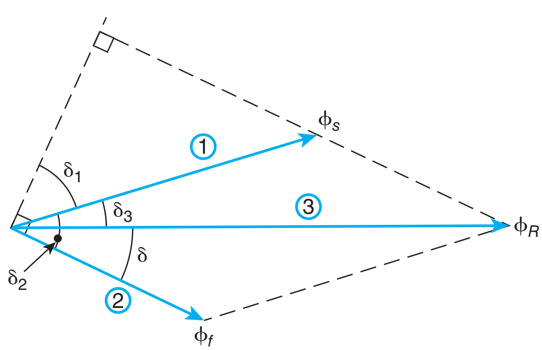

FIG. 3-7 Stator and rotor flux in a 3-ϕ induction machine at standstill.

*For these reasons, three-phase induction motors are sometimes referred to as rotating transformers.

strength of the two fields times the sine of the smallest angle between the two fields. Mathematically,

$$T = K\phi_f\phi_s \sin \delta_2 \tag{3.1a}$$

Since

$$\delta_1 + \delta_2 = 90°$$

Then

$$T = K\phi_f\phi_s \cos \delta_1 \tag{3.1b}$$

where the constant of proportionality K is a function of the physical parameters of the machine, and the factor ($\phi_s \cos \delta_1$) is the quadrature component of the stator flux with respect to the rotor field.

From basic trigonometric concepts, we have

$$\phi_s \cos \delta_1 = \phi_R \cos (\delta_1 + \delta_3) \tag{3.2}$$

and

$$\phi_R \cos (\delta_1 + \delta_3) = \phi_R \sin \delta \tag{3.3}$$

where δ is the phase angle between the resultant field and the rotor field. This angle is called the torque angle. From Eqs. (3.1b), (3.2), and (3.3), we obtain

$$T = K\phi_f \phi_R \sin \delta \tag{3.4}$$

Under ideal conditions, the torque produced in the rotor structure is delivered to the shaft load, while the opposing torque produced in the stator structure is transmitted to the motor's foundation.

3.1.3 Rotating Magnetic Field

This section analyzes the generation of a rotating field by the stator windings of a 3-ϕ induction machine and derives formulas that give the strength and speed of the rotating field.

Consider the elementary two-pole, three-phase machine shown in Fig. 3-8(a). The dots in the center of the three conductors indicate that the current's direction is toward the reader, while the crosses indicate that the assumed positive direction of current is away from the reader. The stator windings carry balanced three-phase currents as shown in Fig. 3-8(b).

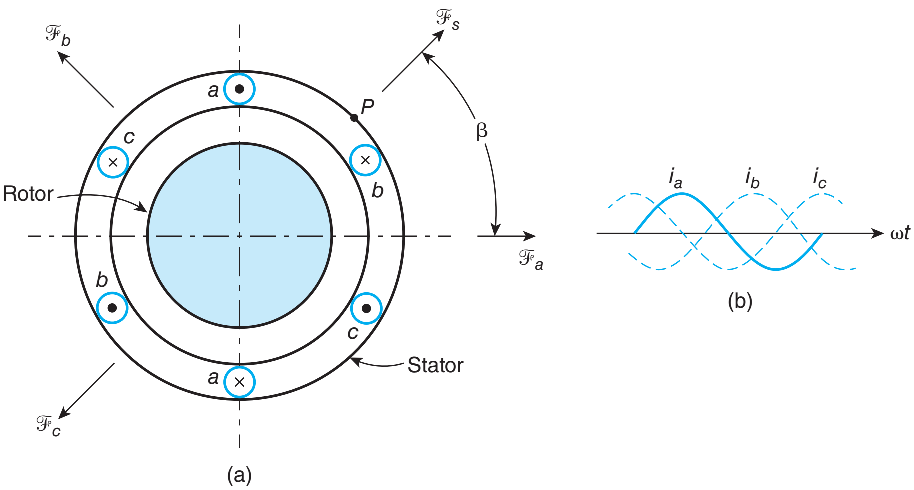

FIG. 3-8 Three-phase induction machine: **(a)** an elementary two-pole induction machine, and **(b)** balanced three-phase stator currents of sequence *ABC*.

For a phase sequence *ABC*, the phase magnetomotive forces (mmf's) as functions of time are as follows:

$$\mathscr{F}_a = N_A I_{m_a} \cos \omega t \tag{3.5}$$

$$\mathscr{F}_b = N_B I_{m_b} \cos (\omega t - 120^\circ) \tag{3.6}$$

$$\mathscr{F}_c = N_C I_{m_c} \cos (\omega t + 120^\circ) \tag{3.7}$$

where I_{m_a}, I_{m_b}, and I_{m_c} are the maximum values of the phase currents, and N_A, N_B, and N_C are the number of turns of the phase windings. The quantity ω is the angular frequency of oscillation of the stator currents, which by definition is

$$\omega = 2\pi f \text{ electrical radians per second} \tag{3.8}$$

where f is the frequency of the stator currents in hertz.

At the instant under consideration, the mmf of phase a coincides with the horizontal axis, as shown in Fig. 3-8(a). The directions of phase mmf's $\mathscr{F}_a$, $\mathscr{F}_b$, and $\mathscr{F}_c$ are obtained by using the right-hand rule.

The resultant stator mmf ($\mathscr{F}_s$) along an axis at an angle β to the horizontal is found by summing up the projections of the phase mmf's along this line:

$$\mathscr{F}_s = \mathscr{F}_a \cos \beta + \mathscr{F}_b \cos (120^\circ - \beta) + \mathscr{F}_c \cos (- 120^\circ - \beta) \tag{3.9}$$

The number of winding turns and the maximum value of the current for each phase are the same. Designating $\mathcal{F}_1$ as the maximum mmf of any one phase, we have

$$\mathcal{F}_1 = N_A I_{m_a} = N_B I_{m_b} = N_C I_{m_c} \tag{3.10}$$

Substituting Eqs. (3.5), (3.6), (3.7), and (3.10) into Eq. (3.9), we obtain

$$\begin{aligned}\mathcal{F}_s = \mathcal{F}_1\,[&\cos \omega t \cos \beta + \cos(\omega t - 120°)\cos(120° - \beta)\\ &+\cos(\omega t + 120°)\cos(-120° - \beta)]\end{aligned} \tag{3.11}$$

By use of the identity

$$\cos x \cos y = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$$

we obtain

$$\begin{aligned}\mathcal{F}_s = \frac{\mathcal{F}_1}{2}[&\cos(\omega t + \beta) + \cos(\omega t - \beta) + \cos(\omega t - 120° + 120° - \beta)\\ &+ \cos(\omega t - 120° - 120° + \beta) + \cos(\omega t + 120° - 120° - \beta)\\ &+ \cos(\omega t + 120° + 120° + \beta)]\end{aligned} \tag{3.12}$$

Simplifying, we get

$$\begin{aligned}\mathcal{F}_s = \frac{\mathcal{F}_1}{2}[&\underline{\cos(\omega t + \beta)} + \cos(\omega t - \beta) + \cos(\omega t - \beta) + \underline{\cos(\omega t + \beta - 240°)}\\ &+ \cos(\omega t - \beta) + \underline{\cos(\omega t + \beta + 240°)}]\end{aligned} \tag{3.13}$$

The sum of the three underlined terms is equal to zero because these phasors are displaced from each other by 120° and because their magnitudes are equal. Therefore, Eq. (3.13) becomes

$$\mathcal{F}_s = \frac{3}{2}\mathcal{F}_1 \cos(\omega t - \beta) \tag{3.14}$$

Equation (3.14) describes a revolving field that rotates counterclockwise with an angular velocity of ω radians per second. The speed of the revolving field is normally designated by ω_s and is referred to as synchronous speed ($\omega_s = \omega$). The flux at a point P that is at β degrees to the horizontal will vary sinusoidally with the same frequency as that of the stator currents.

The revolving field may be visualized as being equivalent to the field generated by a permanent magnet rotated about an axis that coincides with the rotor of the machine.

The effective mmf through the stator structure is equal to 1.5 times the mmf produced by one phase alone. The resulting effective flux, at the absence of saturation and rotor current, is directly proportional to this mmf and inversely proportional to the reluctance of the path through which the flux completes its magnetic circuit.

In general, a rotating field of constant amplitude is produced by an m-phase winding wound $2\pi/m$ electrical radians apart and excited by balanced m-phase currents. The magnitude of the rotating field is $m/2$ times the field produced by any one phase, and its speed of rotation is given by Eq. (3.8). The direction of rotation of the field depends on the phase sequence of the applied currents. When the phase sequence of the supply voltages is reversed, the direction of rotation of the stator field and the speed of the motor are also reversed. The revolving fields of single- and two-phase ac motors are developed in Chapter 4.

Mechanical and Electrical Radians

Consider the elementary four-pole machine shown in Fig. 3-9(a). Starting at point A and going counterclockwise (2π mechanical radians) around the periphery of the machine's stator, the following field polarities are encountered: N_1S_1–S_2N_2–N_3S_3–S_4N_4. This corresponds to two complete electrical cycles (4π electrical radians), as can be seen in Fig. 3-9(b).

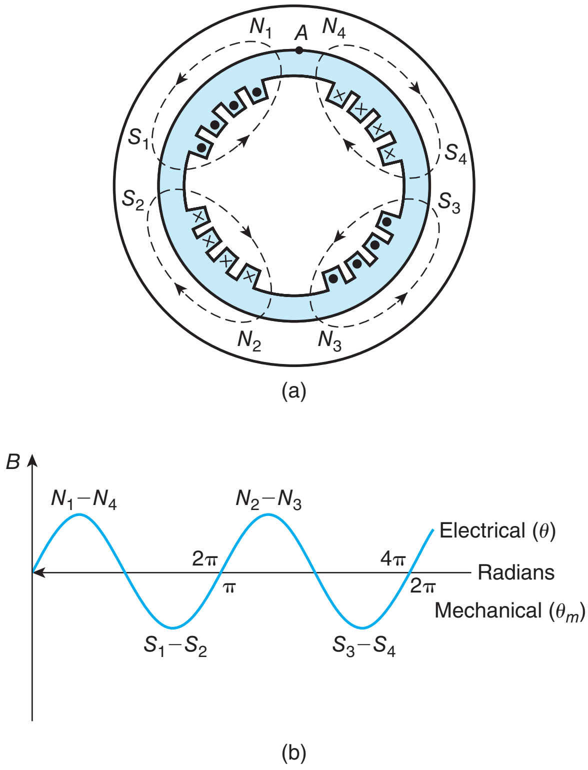

FIG. 3-9 **(a)** Elementary four-pole machine with stator slots and windings omitted; and **(b)** space distribution of field illustrating the relationship between mechanical (θ_m) and electrical (θ) radians.

For a four-pole machine, then, one mechanical or physical revolution will correspond to two complete cycles of the field. In general, for a p-pole machine,

$$\theta = \frac{p}{2}\theta_m \qquad \textbf{(3.15)}$$

where θ and θ_m are the electrical and mechanical radians, respectively.

From Eqs. (3.8) and (3.15), we get the speed of the rotating field in mechanical radians per second:

$$\omega_m = \frac{2}{p}(2\pi f) = 4\pi\frac{f}{p} \text{ mechanical radians/s} \qquad \textbf{(3.16)}$$

Since one revolution is equivalent to 2π mechanical radians, the speed in revolutions per second (n_s) is

$$n_s = 2\frac{f}{p} \text{ r/s} \qquad \textbf{(3.17)}$$

The speed in revolutions per minute is

$$n_s = 120\frac{f}{p} \text{ r/min} \qquad \textbf{(3.18)}$$

Thus, for a 60 Hz system, the synchronous speed of the stator field will be 3600, 1800, or 1200 r/min when the number of poles is two, four, or six, respectively.

EXAMPLE 3-1

For a 480 V, 3-ϕ, four-pole, 60 Hz induction motor, determine the speed of the stator field in:

a. Electrical radians per second.
b. Mechanical radians per second.
c. Revolutions per second.
d. Revolutions per minute.

SOLUTION

a. From Eq. (3.8),

$$\omega_s = 2\pi(60) = \underline{\underline{377 \text{ electrical rad/s}}}$$

b. From Eq. (3.16),

$$\omega_m = 377\left(\frac{2}{4}\right) = \underline{\underline{188.5 \text{ mechanical rad/s}}}$$

c. From Eq. (3.17),

$$n_s = 2\left(\frac{60}{4}\right) = \underline{\underline{30 \text{ rps}}}$$

d. From Eq. (3.18),

$$n_s = 120\left(\frac{60}{4}\right) = \underline{\underline{1800 \text{ r/min}}}$$

Exercise 3-1

Prove that the speed of rotation of the stator field is reversed when the order of rotation of the stator currents is reversed.

3.1.4 Slip

Under normal operating conditions, the rotor rotates in the same direction as the magnetic field of the stator but at a reduced speed. The difference between the synchronous speed of the stator field and the actual rotor speed (n_a) defines the slip (s) of the motor. The slip of a motor is an important parameter, used extensively in the design and analysis of induction machines. The per-unit value of the slip is given by

$$s = \frac{n_s - n_a}{n_s} \tag{3.19}$$

where n_s is the synchronous speed of the rotating field, given by Eq. (3.18).

To achieve higher efficiency, the majority of three-phase induction motors are designed to operate at a very small slip (usually less than 5%) when delivering rated power.

When three-phase induction machines operate as induction generators, their actual rotor speed is higher than their synchronous speed, and their velocity is in the same direction as the synchronously rotating stator field. Thus, their slip is *negative*. Induction generators are used to connect small generating stations to large utility networks. (For a detailed analysis of induction generators, see Section 3.4.2.)

When an induction machine is driven by another motor in such a way that its actual rotor speed is in the direction opposite to that of the synchronously rotating stator field, then n_a in Eq. (3.19) becomes negative, and thus the slip is *greater than unity*. Induction machines operating at a slip greater than unity are used as frequency and voltage multipliers. They are covered in Section 3.4.

EXAMPLE 3-2

A 75 kW, 60 Hz, 480 V, 1176 r/min, three-phase induction motor has, at full-load, a power factor of 80 lagging and an efficiency of 0.90. Under rated operating conditions, determine,

a. The magnitude of the current drawn by the motor.
b. The real, reactive, and complex power drawn by the motor.
c. The output torque at full-load.
d. The operating slip.

SOLUTION

a. By definition,

$$\text{efficiency} = \frac{\text{output power}}{\text{input power}}$$

In mathematical symbols,

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

The output power is 75 kW. The input power is

$$P_{\text{in}} = \sqrt{3}V_{L\text{-}L}I_L \cos\theta$$

From the above, the magnitude of the line current is

$$I_L = \frac{P_{\text{out}}}{\eta(\sqrt{3}V_{L\text{-}L}\cos\theta)} = \frac{75{,}000}{0.90\,(\sqrt{3}(480)(0.80))} = \underline{\underline{125.29\text{ A}}}$$

The current phasor is shown in Fig. 3-10(a).

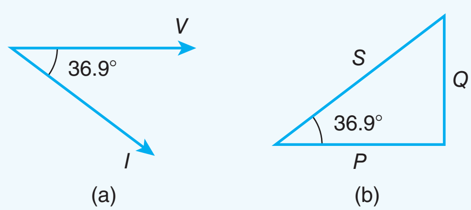

FIG. 3-10

b. The real or consumed power is

$$P_{\text{in}} = \sqrt{3}V_{L\text{-}L}I_L \cos\theta$$

$$= \sqrt{3}(480)(125.29)(0.80) = \underline{\underline{83.33\ \text{kW}}}$$

or

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{75}{0.9} = \underline{\underline{83.33\ \text{kW}}}$$

The reactive power is

$$Q = \sqrt{3}V_{L\text{-}L}I_L \sin\theta$$

$$= \sqrt{3}(480)(125.29)(\sin 36.9^\circ) = \underline{\underline{62.5\ \text{kVAR}}}$$

or

$$Q = P\tan\theta = 83.33 \tan 36.9^\circ = \underline{\underline{62.5\ \text{kVAR}}}$$

The complex power is

$$S = \sqrt{3}V_{L\text{-}L}I^*$$

$$= \sqrt{3}(480)(125.29)\underline{/36.9^\circ} = \underline{\underline{104.17\underline{/36.9^\circ}\ \text{kVA}}}$$

Alternatively,

$$S = P + jQ$$

$$= 83.33 + j62.5 = \underline{\underline{104.17\underline{/36.9^\circ}\ \text{kVA}}}$$

The motor's power triangle is shown in Fig. 3-10(b).

c. The full-load output torque is

$$T = \frac{\text{power}}{\text{speed}} = \frac{75{,}000}{1176\,\frac{2\pi}{60}} = \underline{\underline{609\ \text{N}\cdot\text{m}}}$$

d. By inspection of the given data, it can be determined that the motor's synchronous speed is 1200 r/min. The operating slip is

$$s = \frac{n_s - n_a}{n_s} = \frac{1200 - 1176}{1200} = \underline{\underline{0.02\ \text{per unit (pu)}}}$$

Exercise 3-2

A 480 V, 50 kW, delta-connected three-phase, 60 Hz, six-pole induction motor operates at a slip of 1.5% and has a power factor of 82% and an efficiency of 88%. Determine the following:

a. The magnitude of the line and phase currents.
b. The actual rotor speed.
c. The real, reactive, and complex power drawn by the motor.

Answer (a) 83.34 A, 48.12 A; (b) 1182 r/min;
(c) 56.82 kW, 39.66 kVAR, $69.29\angle 34.9^\circ$ kVA

Exercise 3-3

In a laboratory, a four-pole, 60 Hz induction machine is operated as follows:

a. As an induction motor at a speed of 1760 r/min.
b. As an induction generator at a speed of 1850 r/min.

In each of these three cases, determine the operating slip and the direction of the rotor's rotation relative to the clockwise rotation of the stator field.

Answer (a) 2.22%, cw; (b) −2.78%, cw.

3.1.5 Equivalent Circuit

The analysis of polyphase electric machines is simplified by using their per-phase equivalent circuits. The derivation of an equivalent circuit is based on the consideration of the electromagnetic coupling between the stator and rotor when the rotor is stationary and when it is running.

Rotor Stationary

When the rotor of the motor is stationary, the time rate of change of the flux linkages between the stator and the rotor depends on both the number of winding turns in the rotor and the speed and magnitude of the rotating mmf of the stator. This is similar to the transformer action, where the alternating flux of the primary current sets up flux linkages between the primary and secondary windings. The voltage induced in the secondary winding of a transformer is proportional to the time rate of change of its flux linkages. Similarly, the voltage induced in the rotor winding of a three-phase induction motor is proportional to the time rate of change of the winding's flux linkages.

The frequency (f_r) of the voltage induced in the rotor windings is equal to that of the stator windings. That is,

$$f = f_r \tag{3.20}$$

where f is the frequency of the stator currents.

The per-phase equivalent circuit of a three-phase induction motor at standstill, then, is similar to that of a transformer. Refer to Fig. 3-11(a). The impedance $R_1 + jX_1$ represents the per-phase leakage impedance of the stator windings. The impedance $R_c /\!/ jX_\phi$ represents the per-phase magnetizing impedance of the motor. Because of the air gap between the stator and rotor structures, this

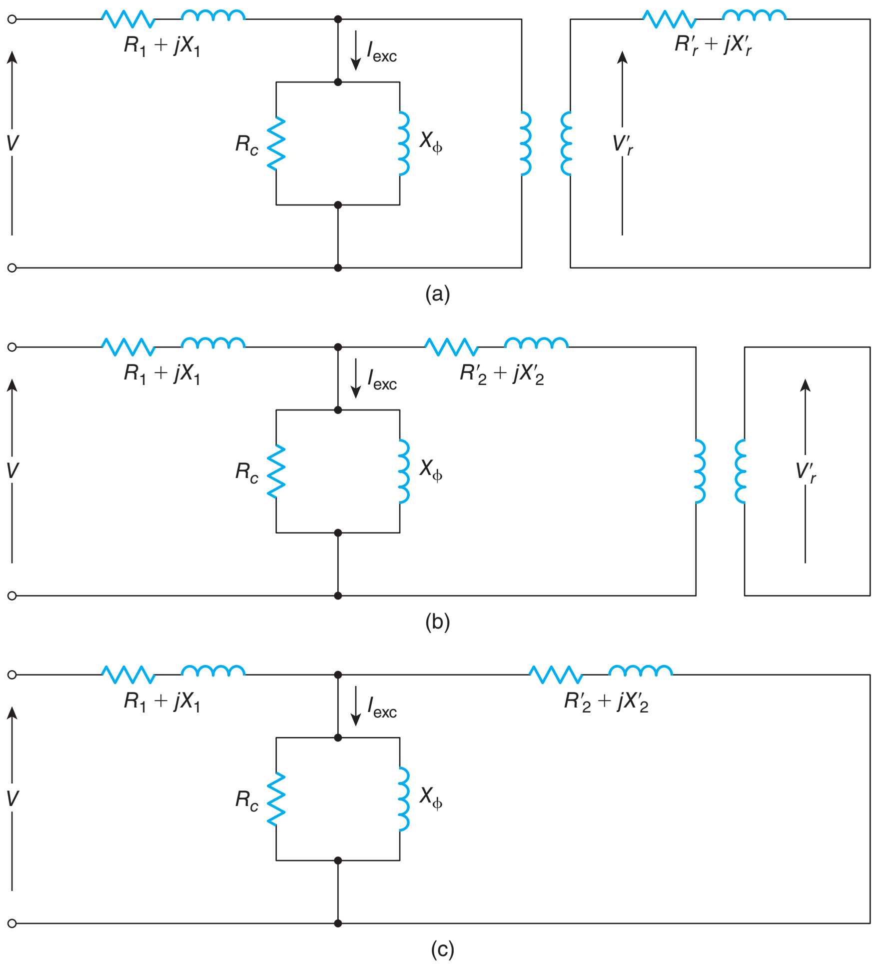

FIG. 3-11 Per-phase equivalent circuit of a three-phase motor at standstill: **(a)** stator and rotor circuits, **(b)** rotor impedance referred to the stator, **(c)** equivalent of (b).

impedance is substantially larger than that of a static transformer of equivalent voltage and volt-ampere rating. The impedance $R'_r + jX'_r$ represents the per-phase leakage impedance of the rotor windings at standstill.

The rotor impedance as seen from the stator (Fig. 3-11(b)) is

$$Z'_2 = a^2(R'_r + jX'_r) \tag{3.21}$$

or

$$Z'_2 = R'_2 + jX'_2 \tag{3.22}$$

where Z'_2 is the rotor impedance at standstill as seen from the stator, and a represents the number of effective turns between the stator and rotor windings.

The rotor windings are shorted, and thus the ideal transformer shown in Fig. 3-11(b) is also shorted. This leads to the per-phase equivalent circuit presented in Fig. 3-11(c). The per-phase input voltage to the stator windings and the excitation current are represented, respectively, by V and I_{exc}.

Rotor Running

When the rotor is running, the time rate of change of the flux linkage between the stator and rotor windings depends on the relative motion between the synchronously rotating stator mmf and the actual rotor speed. Since the relative motion between the stator mmf and the rotor speed is given by the slip, the voltage induced in the rotor windings (V_r) is given by

$$V_r = sV'_r \tag{3.23}$$

where s is the operating slip of the motor and V'_r is the voltage induced in the rotor windings when the rotor is stationary.

The frequency (f_r) of the voltage induced in the rotor windings is related to the frequency (f) of the stator windings by

$$f_r = sf \tag{3.24}$$

Then the per-phase rotor impedance at a slip s is

$$Z_r = R_r + jsX_r \tag{3.25}$$

where R_r and X_r represent the rotor resistance and reactance, respectively. The per-phase equivalent circuit of the motor is shown in Fig. 3-12(a). The rotor current (I_r) is

$$I_r = \frac{V_r}{Z_r} \tag{3.26}$$

$$= \frac{sV'_r}{R_r + jsX_r} \tag{3.27}$$

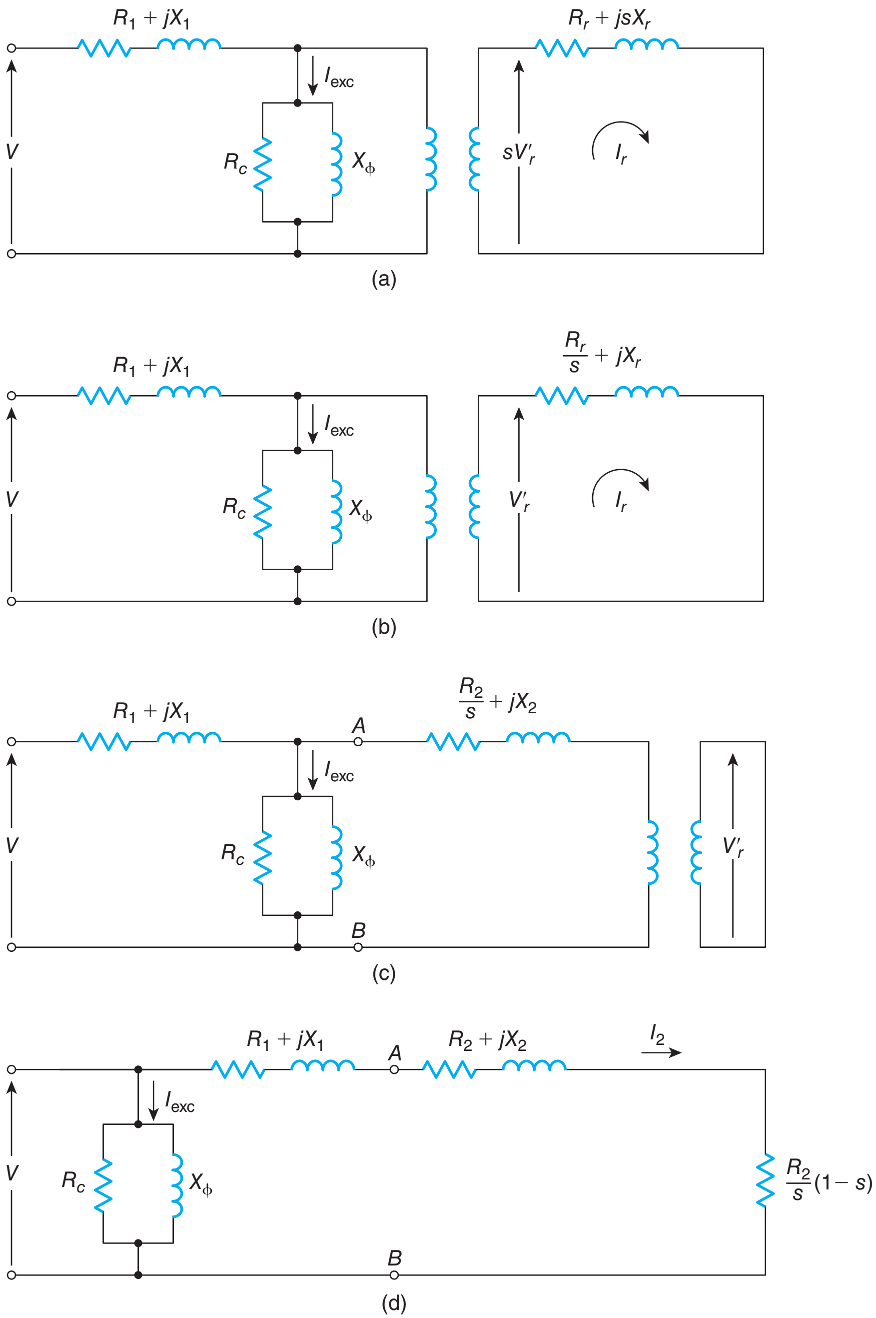

FIG. 3-12 Per-phase equivalent circuit of a three-phase induction motor: **(a)** stator and rotor circuits, **(b)** equivalent of (a), **(c)** equivalent of (b), and **(d)** approximation of (c).

From the above,

$$I_r = \frac{V'_r}{\dfrac{R_r}{s} + jX_r} \tag{3.28}$$

Then the per-phase equivalent circuit of the motor can be drawn as shown in Fig. 3-12(b).

The rotor impedance (Z_2) as seen from the stator is

$$Z_2 = a^2\left(\frac{R_r}{s} + jX_r\right) \tag{3.29}$$

or

$$Z_2 = \frac{R_2}{s} + jX_2 \tag{3.30}$$

The corresponding equivalent circuit is given in Fig. 3-12(c).

The real component of the rotor impedance, as seen by the stator, can also be written as follows:

$$\frac{R_2}{s} = R_2 + R_2\frac{1-s}{s} \tag{3.31}$$

The second term on the right-hand side of this equation is the so-called equivalent mechanical load (R_L) resistance. That is,

$$R_L = \frac{R_2(1-s)}{s} \tag{3.32}$$

By assuming that

$$R'_2 + jX'_2 = R_2 + jX_2 \tag{3.33}$$

then Fig. 3-12(c) represents the general per-phase equivalent circuit of a three-phase induction motor at any speed, as seen from the input terminals of the motor.

When the motor is running, the values of the per-phase rotor resistance (R_r) and inductance (L_r) are different than when their corresponding values are at standstill (R'_r, L'_r) because of higher temperatures, saturation, and the skin effect.

The skin effect (nonuniform current density within the conductors caused by magnetic flux that passes through them and that varies with time) is of particular importance because it causes the rotor's circuit resistance and inductance to vary with the frequency of the rotor currents. This variation is significant during starting, but negligible within the speed range of normal load operation. For accurate results, the actual values of the rotor parameters should be used at starting and running conditions.

In order to further simplify the calculations, the magnetizing branch is usually transferred to input terminals of the stator winding, as shown in Fig. 3-12(d). The magnetizing impedance could also be represented, as in the case of transformers, by its equivalent series-connected components ($R_m + jX_m$). The measurement of the parameters of the equivalent circuit is discussed in Section 3.3.

3.1.6 Torque and Power Relationships

Torque and power relationships will be derived by using the equivalent circuit shown in Fig. 3-12(d). In deriving the general expression for the torque of an induction motor, the effects of the magnetizing impedance will be neglected. Because this magnetizing impedance is relatively large, its effect on the torque developed by the motor at full-load is negligible.

From basic definitions, the per-phase torque developed is

$$T = \frac{\text{power developed}}{\text{actual speed}} = \frac{I_2^2 R_L}{\omega_a} \tag{3.34}$$

The equivalent mechanical load resistance (R_L) is

$$R_L = R_2 \frac{1-s}{s}$$

and the actual speed is

$$\omega_a = \omega_s (1-s)$$

Substituting the above into Eq. (3.34), we obtain

$$T = \frac{I_2^2 R_2 \frac{1-s}{s}}{\omega_s (1-s)} \tag{3.35}$$

$$= \frac{I_2^2 R_2}{\omega_s s} \quad \text{N} \cdot \text{m/phase} \tag{3.36}$$

From the equivalent circuit, the magnitude of the current I_2 is

$$I_2 = \left|\frac{V}{Z}\right| = \frac{V}{\sqrt{\left(R_1 + \frac{R_2}{s}\right)^2 + X_2}} \tag{3.37}$$

where $X = X_1 + X_2$.

From the above, the torque developed in terms of the motor's parameters is

$$T = \frac{V^2}{\left[\left(R_1 + \frac{R_2}{s}\right)^2 + X^2\right]} \left(\frac{R_2}{\omega_s s}\right) \quad \text{N} \cdot \text{m/phase} \tag{3.38}$$

where V is the phase-to-neutral voltage.

The variation of torque versus slip is shown in Fig. 3-13. The abscissa of this curve is the motor's speed or slip. The symbols T_{fl}, T_{st}, and T_m correspond to the full-load, starting, and maximum torque of the motor, respectively. The same diagram also shows the torque-speed characteristic of the mechanical load. The intersection of these characteristics gives the operating torque and speed of the motor. For this reason, it is extremely important that, before selecting a suitable motor, you accurately calculate the torque-speed characteristic of the driven load.

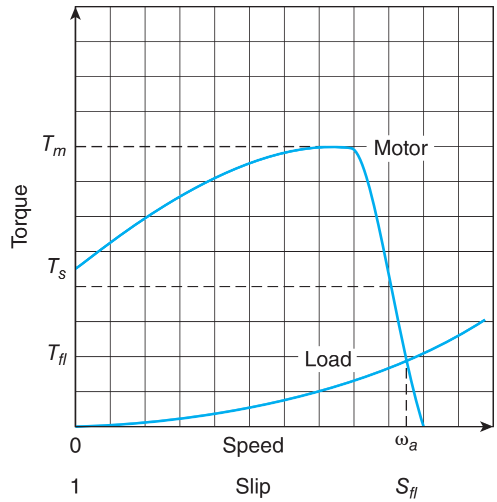

FIG. 3-13 Typical torque-speed characteristics.

The motor and load torque-speed characteristics are also used (as explained on the website) to find the accelerating and decelerating times of the motor. These parameters are important for the transient analysis of machines and for the proper selection of the motor's protective devices.

From Eq. (3.38), it is evident that, for a constant slip, the torque is proportional to the applied voltage squared:

$$T \propto V^2 \tag{3.39}$$

In practice, you may have to evaluate the starting torque capability of a motor because its high starting currents—depending on the impedance of the supply network—reduce the voltage delivered to the stator, and as a result the starting torque is also reduced. When the Thévenin impedance ($R_{th} + jX_{th}$) of the supply network is known, R_1 in Eq. (3.38) should be replaced by $R_1 + R_{th}$, and X should be replaced by $X + X_{th}$.

From Eq. (3.38), after expanding and simplifying, we obtain

$$T = \frac{V^2 s R_2}{\omega_s[(sR_1)^2 + 2sR_1R_2 + R_2^2 + (sX)^2]} \quad \text{N} \cdot \text{m/phase} \tag{3.40}$$

For slips in the full-load range, the predominant term in the denominator of the above equations is R_2^2. Therefore,

$$T \approx \frac{V^2}{\omega_s}\frac{s}{R_2} \tag{3.41}$$

or

$$T \approx K\frac{s}{R_2} \tag{3.42}$$

where K is the constant of proportionality given by

$$K = \frac{V^2}{\omega_s} \tag{3.43}$$

From Eq. (3.42), it is evident that in the full-load operating range, the higher the actual speed of the motor, the lower its output torque. It can also be shown that for a certain range of rotor resistance, the starting torque (T_{st}) of the motor is proportional to its rotor resistance. Mathematically,

$$T_{st} \propto R_2 \tag{3.44}$$

The higher the rotor resistance, the higher the motor's starting torque, and the lower its inrush current. The derivation of Eq. (3.44) is left as a student problem (see Problem 3-2).

Because of the effects of rotor resistance on starting torque and starting current, the high-power motors supplied through weak power systems (i.e., a supply network with relatively high impedance) are of the wound-rotor type.

Maximum or Breakdown Torque

Refer again to the approximate equivalent circuit of Fig. 3-12(d). For maximum power transfer from stator to the rotor-load resistance, the following relationship must be satisfied:

$$|R_1 + jX| = \frac{R_2}{s} \tag{3.45}$$

Equating the magnitudes of this relationship, we get the equation for the slip of the motor at maximum torque (s_{mt}):

$$s_{mt} = \frac{R_2}{\sqrt{R_1^2 + X^2}} \tag{3.46}$$

where

$$X = X_1 + X_2$$

Substituting Eq. (3.46) into Eq. (3.38), after simplification, we find the following expression for the maximum torque developed by the motor:

$$T_m = \frac{V^2}{\left[\left(R_1 + \sqrt{R_1^2 + X^2}\right)^2 + X^2\right]} \times \frac{\sqrt{R_1^2 + X^2}}{\omega_s} \ \text{N} \cdot \text{m/phase} \tag{3.47}$$

Thus, the maximum torque is independent of rotor resistance. The rotor resistance controls the slip or the speed where the maximum torque occurs. Typical torque-speed characteristics, as a function of rotor resistances, are shown in Fig. 3-14.

With negligible stator resistance ($R_1 = 0$), it can be shown that the torque (T) at any slip (s) is related to the maximum torque (T_m) and its corresponding slip (s_{mt}) by

$$\frac{T}{T_m} = \frac{2ss_{mt}}{s_{mt}^2 + s^2} \tag{3.48}$$

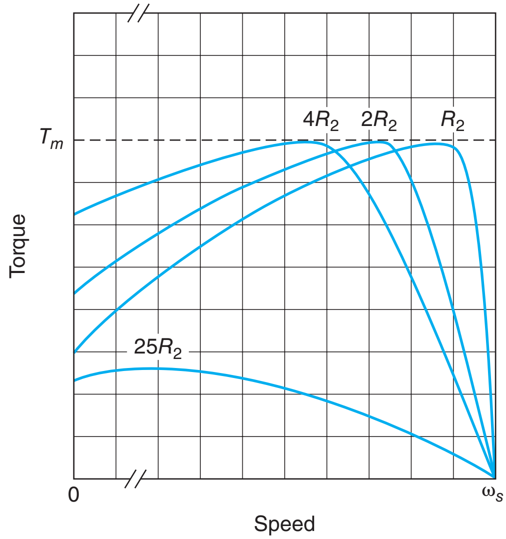

FIG. 3-14 Torque-speed characteristics as a function of rotor resistance.

If the torque-speed characteristic of the motor is not available, then the above equation can be used, in conjunction with some of the motor parameters (T_{st}, T_m, T_{fl}, s_{fl}) to sketch it. The derivation of Eq. (3.48) is left as a student problem (see Problem 3-2).

The torque developed by a motor must be sufficient to provide the motor's rotational losses and the torque requirements of the mechanical load. The torque requirements of a mechanical load may include any or all of the following torque components: the torque component $[(J(d\omega_a/dt)]$ required to accelerate the rotating masses up to the steady-state speed; the torque component ($B\omega_a$) that represents the viscous friction of the rotating parts; and any other torque (T_L) requirements of the load. In mathematical form,

$$T = J\frac{d\omega_a}{dt} + B\omega_a + T_L \tag{3.49}$$

where

J = the polar moment of inertia of the rotating masses in Newton-meter-second squared
B = the coefficient of viscous friction in $\text{N} \cdot \text{m/(rad/s)}$
T_L = a component of load torque in $\text{N} \cdot \text{m}$
ω_a = the actual speed of the motor in rad/s.

Equation (3.49) is derived from Newton's second law of motion for rotating bodies. This law states that the sum of the torques ($\Sigma\, T$) about an axis of rotation is equal to the polar moment of inertia (J) times the angular acceleration (α) of the rotating masses. Mathematically,

$$\Sigma\, T = J\alpha \tag{3.50}$$

This law is of primary importance to the dynamic analysis of all machines, and its applications are discussed on the accompanying website.

In developing the torque relationships, the magnetizing impedance of the motor was neglected in order to simplify the calculations. The magnetizing impedance, as already mentioned, does not affect the torque of the motor at full load; however, at light loads, it has significant effects. For accurate torque calculations, the magnetizing impedance should be taken into consideration. You can easily do so by replacing, in Eqs. (3.38) and (3.47), the voltage source (V) and stator impedance (Z_1) by the equivalent Thévenin's voltage and impedance. Referring to Fig. 3-12(c), and looking toward the input from terminals A–B, we obtain

$$V_{th} = \frac{V(R_c /\!/ jX_\phi)}{R_1 + jX_1 + R_c /\!/ jX_\phi} \tag{3.51}$$

and

$$Z_{th} = (R_1 + jX_1) /\!/ (R_c /\!/ jX_\phi) \tag{3.52}$$

From Eqs. (3.37) and (3.46), we find that the current I at a slip s is related to the current at maximum torque (I_{mt}) and its corresponding slip (S_{mt}) by

$$\left(\frac{I}{I_{mt}}\right)^2 = \frac{2s^2}{s_{mt}^2 + s^2} \tag{3.53}$$

The equations developed in this section are based on the approximate equivalent circuit of Fig. 3-12(d). They are *not* applicable to motors with deep-bar rotor construction or to double squirrel-cage rotors. In such cases, the proper equivalent circuit should be drawn, then from basic principles, the desired relationships can be derived.

Power Considerations

The power across the air gap (P_{ag}) provides the power developed and the rotor copper loss. That is,

$$P_{ag} = I_2^2 R_2 \frac{1-s}{s} + I_2^2 R_2 \tag{3.54}$$

Thus,

$$P_{ag} = I_2^2 \frac{R_2}{s} \text{ W/phase} \tag{3.55}$$

As Eq. (3.55) shows, the rotor-winding losses are directly proportional to the air-gap power and the operating motor slip. Thus, the smaller the slip, the lower the rotor copper losses. For this reason, the slip, at full-load, is usually kept below 5%.

The power developed (P_d) furnishes the output power as well as the rotational losses. Mathematically,

$$P_d = I_2^2 R_2 \frac{1-s}{s} \text{ W/phase} \tag{3.56}$$

$$= P_{\substack{\text{rotational}\\\text{losses}}} + P_{\text{output}}$$

The rotational losses include windage, friction, and stray load losses.

Example 3-3

Figures 3-15(a) and (b) show, respectively, the one-line diagram of a three-phase induction motor and its per-phase equivalent circuit referred to the stator. Assuming constant rotor impedance, and neglecting the effects of the magnetizing current and rotational losses, determine:

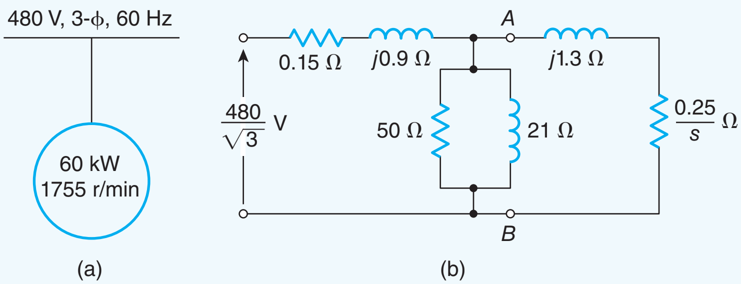

FIG. 3-15

a. The number of poles.
b. The synchronous speed of the stator field.
c. The per-unit slip.
d. The line current at starting.
e. The starting torque.
f. The full-load current, power factor, torque, and efficiency.
g. The slip and current at maximum torque.
h. The maximum torque.

SOLUTION

a. Under normal operating conditions, the slip of a three-phase induction motor is kept small because of its direct relationship to the copper losses. Thus, the actual speed of the motor should be very close to the speed of the synchronously rotating stator field. Since the actual speed is given as 1755 r/min, the closest synchronous speed is 1800 r/min. From Eq. (3.18), the number of poles is

$$p = \frac{120 \times 60}{1800} = \underline{\underline{4 \text{ poles}}}$$

b. The speed of the stator field is

$$n_s = \underline{\underline{1800 \text{ r/min}}}$$

or

$$\omega_s = 1800\,\frac{2\pi}{60} = \underline{\underline{188.5 \text{ rad/s}}}$$

c. Using Eq. (3.19), we find that the slip in per unit is

$$s = \frac{1800 - 1755}{1800} = \underline{\underline{0.025 \text{ pu}}}$$

d. The line current at starting is found by applying Ohm's law in the equivalent circuit of Fig. 3-15(c):

$$I_{st} = \frac{480/\sqrt{3}}{0.15 + 0.25 + j2.2} = \underline{\underline{123.94\,\angle{-79.7^\circ}\text{ A}}}$$

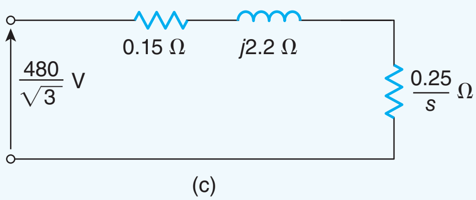

FIG. 3-15(c)

e. Since the motor is star-connected and the magnetizing impedance is neglected, the rotor current as seen from the motor terminals is equal to the stator or line current. Substituting the known data into Eq. (3.36) gives the motor's starting torque:

$$T_{st} = 3(123.94)^2\,\frac{(0.25)}{188.5} = \underline{\underline{61.12 \text{ N}\cdot\text{m}}}$$

f. The current at full-load is

$$I_{fl} = \frac{480/\sqrt{3}}{0.15 + \dfrac{0.25}{0.025} + j2.2} = 26.68\,\angle{-12.2^\circ}\text{ A}$$

By definition, the power factor is cos 12.2° = 0.977 lagging. The power factor is unrealistically high because the magnetizing impedance was assumed to be negligible.

The full-load torque is

$$T_{fl} = \frac{3(26.68)^2}{188.5}\left(\frac{0.25}{0.025}\right) = \underline{\underline{113.32 \text{ N} \cdot \text{m}}}$$

The efficiency of the motor at full-load is

$$\eta = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}} = 1 - \frac{3I^2R}{\sqrt{3}V_{L\text{-}L}I_L \cos\theta} = 1 - \frac{3(26.68)(0.40)}{\sqrt{3}(480)(0.977)}$$

$$= \underline{\underline{0.96}}$$

g. From Eq. (3.46), the slip at maximum torque is

$$s_{mt} = \frac{0.25}{\sqrt{0.15^2 + 2.2^2}} = \underline{\underline{0.11 \text{ pu}}}$$

The corresponding motor's speed is

$$\eta_a = 1800(1 - 0.11) = 1596 \text{ r/min}$$

The current at maximum torque is

$$I = \frac{480/\sqrt{3}}{0.15 + \dfrac{0.25}{0.11} + j2.2} = \underline{\underline{85.99 \angle -43^\circ \text{ A}}}$$

h. The maximum torque is

$$T_m = \frac{3(85.99)^2(0.25)}{188.5(0.11)} = \underline{\underline{259.44 \text{ N} \cdot \text{m}}}$$

For purposes of comparison, the results are summarized in Table 3-1.

TABLE 3-1 Summary of the results of Example 3-3

Operating Condition	Speed in r/min	Current in A	Power Factor Lagging	Torque in N · m
Starting ($s = 1$)	0	123.94	0.18	61.12
Full-load ($s = 0.025$)	1755	26.68	0.977	113.32
Maximum torque ($s_{mt} - 0.11$)	1596	85.99	0.73	259.44

Exercise 3-4

The per-phase equivalent circuit of a 480 V, 60 Hz, 1176 r/min induction motor is shown in Fig. 3-16. Neglecting the magnetizing impedance, determine the starting, full-load, and maximum torque of the motor.

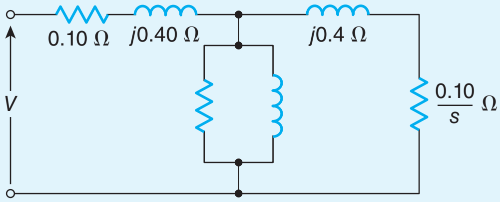

FIG. 3-16

Answer 269.63 N · m, 343.99 N · m, 1011.6 N · m

3.2 Industrial Considerations

This section discusses the industrial classification of squirrel-cage induction motors and mechanical loads, the change in a motor's input parameters that accompany variations in the load requirements, a motor's efficiency and power factor, and the effects of the supply voltage on the torque-speed characteristic.

3.2.1 Classification of Induction Motors

The National Electrical Manufacturers Association (NEMA) has developed a code-letter system by which a letter (A, B, C, or D) designates a *particular class of motors* with specific characteristics. The various characteristics are mainly obtained by a unique rotor design. Table 3-2 shows the general characteristics and the corresponding applications of each class of motor. Typical torque-speed characteristics for the various code-letter designations of squirrel-cage induction motors are shown in Fig. 3-17. Before a motor is selected, the characteristics of the load must be known. The load characteristics include power requirements, starting and full-load torque, speed variation, acceleration characteristics, and the environment in which the motor is to operate.

To select a particular type of motor for a given mechanical load, the torque-speed characteristic of the motor must be compared to that of the mechanical load. The intersection of the motor-load characteristics must correspond not only to the speed and torque requirements of the load, but also to the highest efficiency for the motor. The *starting torque* of the motor must be larger than the

TABLE 3-2 Characteristics and applications of 3-ϕ Induction Motors

Design	Starting Torque, in Per Unit	Starting Current, in Per Unit	Maximum Torque, in Per Unit	Range of Full Load Slip	Applications
A	1.5 to 1.75	5 to 8	2 to 25	2% to 5%	General-purpose motors, fans, blowers, most machinery tools
B	1.5 to 1.75	4.5 to 5	2 to 3	3% to 6%	Same as Class A
C	2 to 2.5	3.5 to 5	1.9 to 2.25	4% to 8%	Used in conveyors, compressors
D	2.75 to 3	3 to 8	2.75	7% to 17%	Used for high-inertia loads, low efficiency

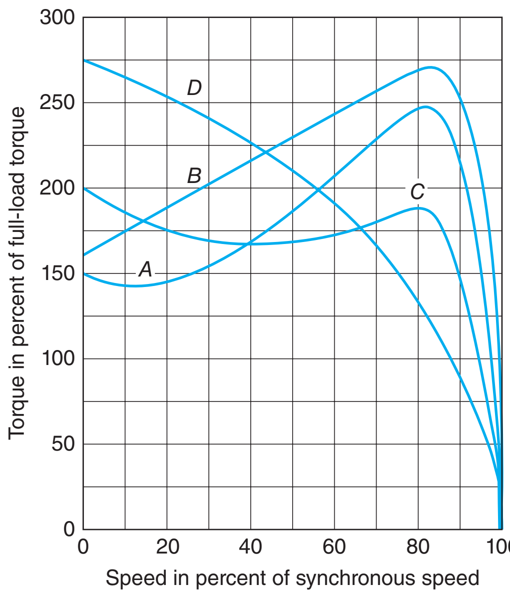

FIG. 3-17 Typical torque-speed characteristics of squirrel-cage induction motors.

load requirement, or the motor will not be able to start rotating. As a result, it will draw its locked-rotor current, perhaps damaging the motor.

The minimum value of the torque in the region between starting and maximum torque is called the *pull-in torque*. As can be seen from the general torque-speed characteristics (Fig. 3-17), only class A and C motors have a meaningful pull-in torque of the motor, which must be lower than that of the motor itself; otherwise, the rotor will not accelerate smoothly.

The *maximum torque* developed by a motor indicates the capability of the machine to overcome high transient-load torques. A motor with a maximum torque that is relatively low may stall when a sudden load torque exceeds the motor's breakdown torque.

The *accelerating torque* of a motor can be obtained from its torque-speed characteristics by comparing the differences in the areas under the torque-speed curves of the motor and the load. (For more detail, see Chapter 3W on the accompanying website.)

A motor's accelerating time (the time it takes a motor, starting from rest, to reach its steady-state speed) is inversely proportional to its accelerating torque. For all practical purposes, during acceleration, a motor draws locked-rotor current. The longer a motor's accelerating time, the greater the possibility of thermal damage to its windings. For this reason, the maximum number of permissible starts during a specific time interval should be carefully determined, and the motor's manufacturer should—as is usually the practice—be consulted.

For any given mechanical load, a class D motor is best equipped to reduce acceleration time (see Fig. 3-17). The cost of a motor, however, increases progressively from class A to class D.

3.2.2 Mechanical Loads

The selection of a particular motor for a specific load requirement is based on the torque-speed characteristic of the mechanical load. Manufacturers of motors will provide the characteristics of a motor: its starting current and torque, maximum torque, starting and full-load power factors, and so on. Similarly, manufacturers of fans, pumps, and the like can provide the torque-speed characteristics of their equipment.

When the torque-speed characteristic of a particular load is not available, it can be derived by drawing the free-body diagram of the system under consideration and by applying Newton's laws of motion.

Load Classification

Mechanical loads are classified according to their torque-speed characteristics. There are actually many such characteristics, but all may be grouped into one of four general categories: constant torque; torque proportional to speed; torque inversely proportional to speed; and torque proportional to the square of the speed (see Fig. 3-18). The mathematical descriptions and the actual torque-speed characteristics of mechanical loads are usually evaluated by mechanical process engineers and can generally be found in standard mechanical engineering handbooks.

Constant Torque

A constant-torque load is a load whose torque remains constant within the operating speed range of the load. Examples of constant-torque loads are conveyors, grinding mills, and crane-hoist systems.

Torque Proportional to Speed

A load whose torque requirement is proportional to its speed is the calender—a machine used for pressing and smoothing cloth or paper between rollers. The smoothness of this page depends on the calender of the paper machine that produced it.

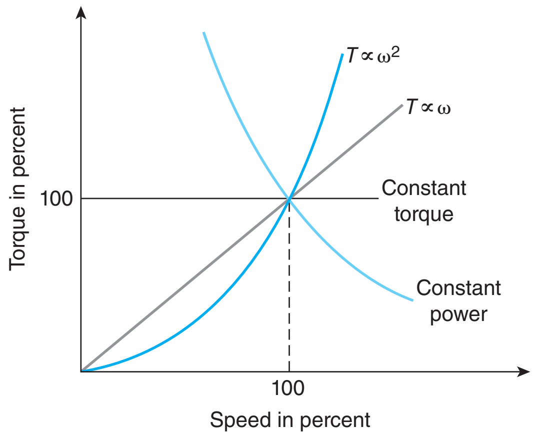

FIG. 3-18 General torque-speed characteristics of mechanical loads.

Torque Inversely Proportional to the Speed

A load whose torque is inversely proportional to its speed is also referred to as the constant-power load. Examples of constant-power loads are circular saws and lathe drives.

Torque Proportional to the Square of the Speed

Examples of loads whose torque requirements are proportional to the square of their speed (power is proportional to the third power of the speed) are centrifugal fans and blowers. All the loads of air-handling fans used in heating, ventilating, and air conditioning systems (HVAC) are of this type.

Exercise 3-5

Prove the following:

a. The mechanical load on a motor that drives a crane hoist is of the constant torque type. (*Hint*: Torque is equal to the force times its moment arm.)

b. The torque of a motor that drives a fan in an air-handling system is directly proportional to the square of the motor's speed. (*Hint:* From the energy equation of hydraulics, the frictional losses are proportional to the square of the speed of the air flow.)

c. The torque of a motor that drives rollers between which paper is pressed and smoothed is directly proportional to the speed of the rollers.

d. The torque required by a circular saw is inversely proportional to its speed.

3.2.3 Mechanical Load Changes and Their Effects on a Motor's Parameters

This section details how changes in load requirements affect the current, power factor, and efficiency of a motor. A load characteristic may be changed when the dampers of a ventilating fan are adjusted, for example, or when a pump's control valve is reset. Such adjustments may be necessary in order to satisfy a particular rate-of-flow condition, which in turn enables the successful completion of a process.

Figure 3-19 shows the torque-speed characteristics of a motor and of a variable mechanical load. Under *nominal* operating conditions, the speed and the torque developed by the motor are given by the intersection of the motor and load characteristics, identified by the symbol Q_1. Under this load requirement, the motor operates at full-load condition. Its losses (copper and rotational) are minimum in comparison to the output power; its stator and rotor currents are of nominal magnitude, and thus the motor—all other conditions being equal—will operate satisfactorily during its entire lifetime. The motor's power factor, efficiency, torque, and speed under nominal operating conditions are all inscribed on its nameplate.

When the torque requirements of the load are *reduced*, the new operating point is given, as before, by the intersection of the torque-speed characteristics. This point is identified by Q_2. As shown, the speed of the motor will increase and its slip will be reduced. Then the impedance* of the motor, as can be seen from the motor's equivalent circuit, will also increase. As a result, the current—at constant terminal voltage—will be reduced.

The reduction in the stator current will reduce the copper losses of the motor but will not appreciably affect the rotational losses. Although the total losses

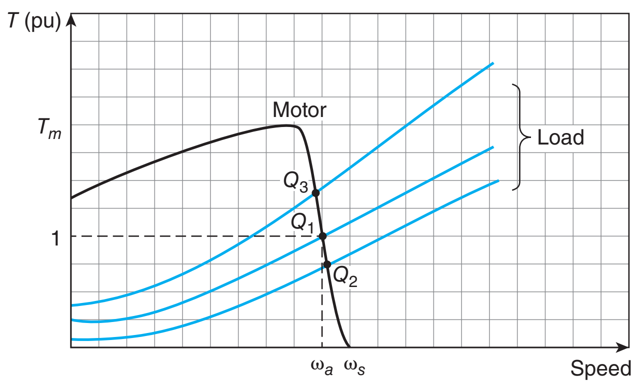

FIG. 3-19 Torque-speed characteristics of an induction machine and its mechanical load.

$*Z = R_1 + (R_2/s) + jX$ ohms.

decrease—as does the output power—the efficiency of the motor also *decreases*. The power factor will increase because the reduction in slip increases the real component of motor impedance. The motor, as seen from its supply lines, appears more resistive.

When the torque requirements of the load *increase*, the motor's torque-speed characteristic is shifted upward. The new operating point of the motor-load system is given by Q_3. The speed of the motor is reduced, and its torque is increased in order to satisfy the new load requirements. The slip of the motor increases [see Eq. (3.19)], and as a result, the impedance of the motor decreases and becomes more inductive. This increase in the operating slip is accompanied by an increase in the current drawn by the motor; consequently, the motor's copper losses increase. The rotational losses remain about constant, and the efficiency of the motor normally decreases, although the output power of the motor increases.

Exercise 3-6

A six-pole, 60 Hz, 3-ϕ induction motor drives a ventilating fan. The motor torque at full-load, as a function of slip, is

$$|T_{mfL}| = 4000s \text{ N} \cdot \text{m}$$

The torque of the load as a function of the speed is

$$T_L = 10 + 4.616 \times 10^{-3}\, \omega^2 \text{ N} \cdot \text{m}$$

a. Sketch the torque-speed characteristics of the motor and load, and determine the torque and power of the motor.
b. Repeat (a), given that the dampers of the ventilating system are adjusted so that the load characteristic becomes

$$T_L = 10 + 16.484 \times 10^{-3}\, \omega^2 \text{ N} \cdot \text{m}$$

Answer (a) 80 N · m, 9.85 kW; (b) 240 N · m, 28.35 kW

3.2.4 Voltage, Efficiency, and Power-Factor Considerations

Voltage Considerations

Induction motors whose nominal capacity is less than 200 kW are designed to operate at voltages less than 1 kV. These are referred to as low voltages (LV). In contrast, motors with a power rating of more than 200 kW are generally designed to operate at voltages within the 1 kV to 15 kV range. These are referred to as medium voltages (MV). The most popular MV levels are 2.3 kV, 4.16 kV, 6.6 kV, and 13.2 kV. The 4 MW grinding mills used in the cement industry, for example, are rated at 4.16 kV, while the 8 MW motors used in the pulp and paper industry are designed to operate at 13.2 kV. MV induction machines are certainly more expensive than LV motors of the same kW rating, but they draw reduced inrush and full-load currents, require less expensive cables, and decrease the losses throughout the upstream electrical network.

In general, the torque developed by a motor at a given speed is proportional to the square of the applied voltage, as demonstrated by Eq. (3.38). Refer to Fig. 3-20. For a given voltage, while all other parameters remain constant, the motor's torque-speed characteristic is fixed. Changes in the supply voltage will change the motor's characteristic accordingly. By reducing the voltage from V' to V volts, the motor's torque-speed characteristic is shifted downward. As a result, the motor's torque—at any speed—is reduced.

For small variations in the load requirements and for a constant ac voltage supply, the motor's *speed variation is limited*. A moderate reduction in the speed will be accompanied by a drastic increase in the current drawn by the motor (see manufacturer's data, Fig. 3-23). If the current protective devices do not remove the motor from its supply voltage, it will overheat. For a given mechanical load, on the other hand, a substantial increase in the speed of the motor is not possible. Under normal conditions, the motor operates very close to synchronous speed, regardless of supply voltage. This effect is evident from the torque-speed characteristics of any 3-ϕ induction motor.

Thus, squirrel-cage induction machines supplied through a constant ac voltage source have a very small speed-variation range; consequently, they cannot be used for mechanical loads that require even a $\pm 2\%$ variation in speed. Another disadvantage of the constant ac voltage supply is that the inrush current of the motor—about six times its nominal value—may cause a significant voltage drop through the upstream impedances. As a result, the motor's *starting torque* is reduced, and so the motor may not be able to start rotating its mechanically coupled load.

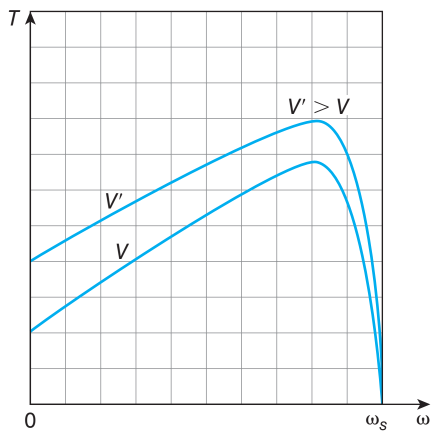

FIG. 3-20 Torque-speed characteristics as a function of voltage.

Furthermore, large inrush currents produce excessive heat through the rotor-stator structure, so successive starts within seconds or even within minutes may damage the motor's windings. For this reason, large motors that draw intolerably high starting currents are provided with controls—see Section 3.5—through which their starting current is reduced during the starting of the machine.

Exercise 3-7

Under normal operating conditions, the windings of a 3-ϕ induction motor are connected in delta (Δ); at starting, they are connected in star (Y). Determine the motor's torque and line current at starting in per unit.

Answer Answer $T = 0.33$ pu

Efficiency Considerations

As illustrated in Fig. 3-21, the efficiency of the motor depends on its speed, its power rating, and the driven load. These values are all normally selected by mechanical process engineers, and not by electrical engineers, who handle all other aspects of the motor—its specifications, purchasing, protection, and so on.

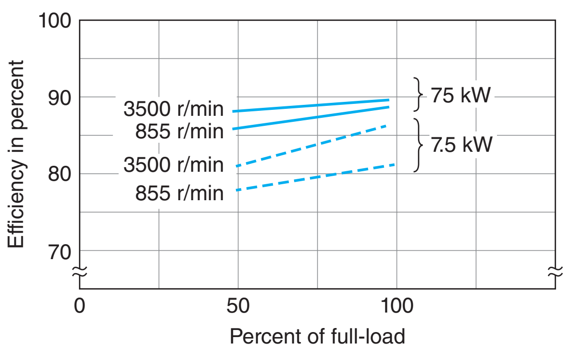

FIG. 3-21 Efficiency of 480 V, squirrel-cage induction motors as a function of load, speed, and nominal power.

In recent years, owing to continuously rising energy costs, two types of induction machines have been made available: those with "standard efficiency" and those with "high efficiency." High-efficiency machines are more expensive than standard-efficiency machines, but in the long run, thanks to their lower energy losses, they are more economical. The efficiencies of these two types of motors and their approximate costs are shown in Table 3-3.

Efficiency measurements may differ from manufacturer to manufacturer, especially when motors are built overseas. In evaluating the advantages of higher-efficiency machines, then, it is important to know how these efficiency ratings are determined.

TABLE 3-3 Squirrel-cage, 480 V, 3-ϕ, four-pole induction machines: typical "standard" and "high" efficiencies and their corresponding costs.

Rating in kW	Efficiency in Percent		Cost Per Unit*	
	Standard	High	Standard Efficiency	High Efficiency
7.5	85.9	90.2	1	1.4
50	91.3	95	8	9.8
75	93	95	10	12
100	95	95.8	16	19

*The cost per unit, in 2010 dollars, is $500.
Based on data from General Electric Canada, Inc.

Power-Factor Considerations

As illustrated in Fig. 3-22, the power factor of the motor depends on its speed, its kW rating, and the operating load. When the power factor of a plant is low, it can be improved by installing sufficient capacitor banks. Selecting a capacitor bank requires consideration and analysis of the following:

- Optimum kVAR and voltage ratings.
- Optimum connection within the power distribution network.
- Resonance conditions and the effects of harmonics.

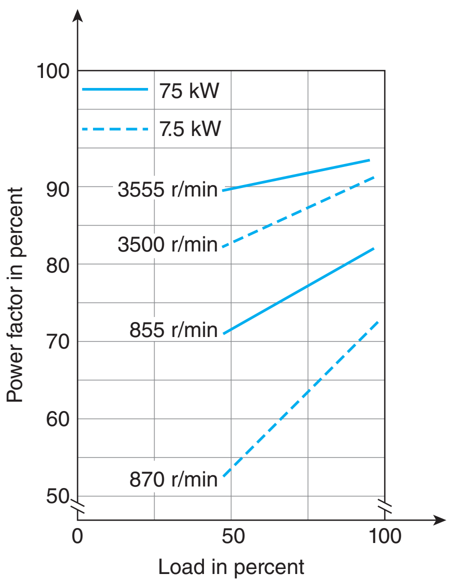

FIG. 3-22 Power factor of 480 V induction machines as a function of load, speed, and nominal power.

When a motor and its power-factor improvement capacitor are switched through a common disconnect device, the rating of the capacitor must be such as not to magnetize the motor each time the motor is switched OFF. In order to avoid this condition, the rated current of the capacitor must be less than the reactive no-load magnetizing current of the motor. If the capacitor bank is oversized, it may, during motor stopping, magnetize (source of excitation) the stator windings. Therefore, owing to the motor's decelerating rotor (source of rotation), the motor may operate as a generator. This condition may be accompanied by high transient voltages and torques, and as a result, the normal life expectancy of the motor may be shortened.

Exercise 3-8

Explain the following:

a. The higher the nominal speed of a motor, the higher its power factor and efficiency.
b. The designing of a plant with a low power factor is due to oversized equipment.

Actual Motor Characteristics

Figure 3-23 shows the torque, current, and power factor versus speed characteristic for a particular 2400 kW, 2300 V, 3-ϕ, 60 Hz, squirrel-cage induction machine. It should be noted that a small decrease in the operating speed will be accompanied by a large increase in the current. For this reason, the characteristics of the mechanical load should be accurately evaluated; otherwise, the motor will operate at larger slips and thus higher motor currents. In practice, all ac machines can withstand a continuous current overload of 10% without overheating.

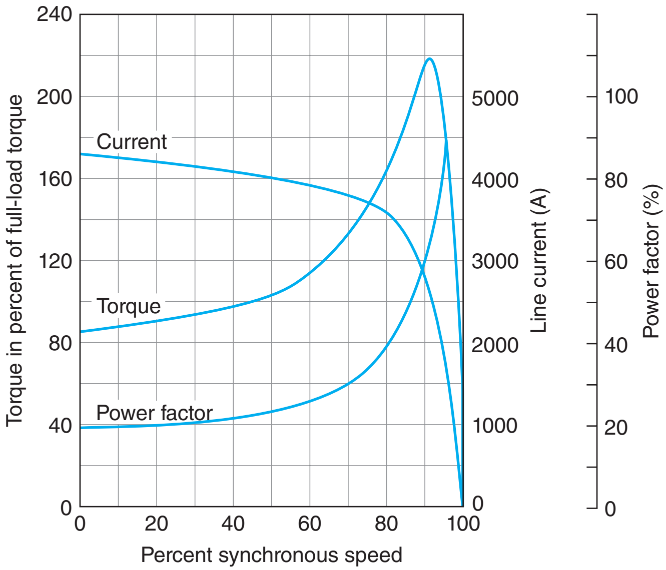

FIG. 3-23 Output torque, line current, and power factor versus speed for a four-pole, 60 Hz, 2400 kW, 2300 V squirrel-cage induction motor. Rated speed: 1765 r/min; rated current: 705 A; power factor: 90%. *Based on data from Westinghouse Canada Inc.*

EXAMPLE 3-4

The three-phase induction motor shown in the one-line diagram of Fig. 3-24(a) has the following starting characteristics:

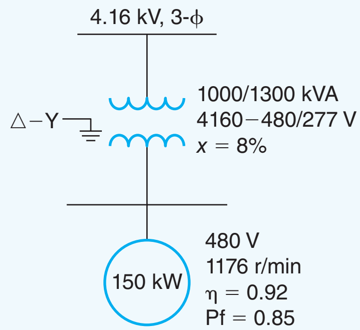

FIG. 3-24(a)

$$T_{st} = 2.2 \text{ pu}, \quad I_{st} = 6\angle{-75^\circ} \text{ pu}$$

The motor receives its voltage supply through a transformer whose nameplate data are shown on the diagram. Neglecting the impedance of the cables and that of the 4.16 kV source, determine, at motor starting:

a. The voltage at the motor terminals.
b. The starting torque of the motor in per unit.
c. The size of the capacitor bank that, when connected in parallel to the motor, will minimize the current through the upstream network.
d. The torque developed by the motor after the capacitor bank has been added.

SOLUTION

The magnitude of the rated current of the motor is

$$I = \frac{P}{\sqrt{3}V_{\text{L-L}} \cos\theta(\eta)} = \frac{150{,}000}{\sqrt{3}(480)(0.92)(0.85)} = 230.72 \text{ A}$$

The magnitude of the starting current of the motor is

$$I_{st} = 230.72(6) = 1384.31 \text{ A}$$

In phasor form,

$$I_{st} = 1384.31\angle{-75^\circ} \text{ A}$$

Taking the nameplate data of the transformer as base parameters, we have

Base impedance:

$$X_{bL} = \frac{V_b^2}{S_b} = \frac{(0.48)^2}{1.0} = 0.23\ \Omega/\text{phase}$$

Transformer reactance:

$$X_{aL} = X_{bL}\,(X_{\text{pu}}) = 0.23(0.08) = 0.018\ \Omega/\text{phase}$$

a. From KVL (Fig. 3-24(b)), the terminal voltage across the motor at starting is

$$V = V_s - IZ$$

$$V\underline{/0^\circ} = \frac{480}{\sqrt{3}}\underline{/\theta_1} - 1384.31\underline{/-75^\circ}\,(0.018\underline{/90^\circ})$$

from which

$$\theta_1 = 1.4^\circ$$

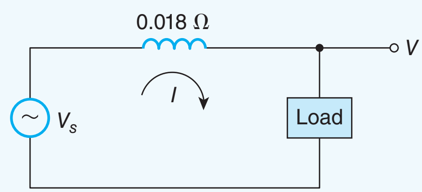

FIG. 3-24(b)

and

$$V = 252.40\ \text{V/phase}$$
$$= \underline{\underline{437.18\ \text{V, L-L}}}$$

Since the motor's terminal voltage is reduced from 480 V to 437.18 V, the starting current must also be proportionately reduced. By repeating the calculations with a new starting current, one will find that the motor's terminal voltage remains close to 437 V.

b. The torque developed by the motor is proportional to the square of the voltage. That is,

$$T \propto V^2$$

from which

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^2$$

where subscript 1 identifies the parameters at starting, as given by the data of the motor, and subscript 2 identifies the corresponding motor parameters at the actual starting conditions. Substituting the known values, we obtain

$$T_{st} = 2.2\left(\frac{437.18}{480}\right)^2 = \underline{\underline{1.82 \text{ pu}}}$$

The starting-torque capability of the motor has been reduced from 2.2 pu to 1.82 pu, owing to the voltage drop on the upstream network. If the driven mechanical load requires a starting torque of 2.0 pu, for example, the motor will not be able to start rotating.

c. One method of reducing the voltage drop at starting is to connect a capacitor bank in parallel to the motor. The capacitor bank must be of a magnitude that will cancel the reactive component of the starting current. Thus, the capacitor's current (I_c) should be

$$I_c = 1384.31 \sin 75° = 1337.14 \text{ A}$$

The size of the capacitor bank is

$$\begin{aligned} Q &= \sqrt{3} V_{\text{L-L}} I \sin\theta \\ &= \sqrt{3}(480)(1337.14)(1) \\ &= 1111.68 \text{ kVAR*} \end{aligned}$$

d. The upstream current is

$$\begin{aligned} I &= I_m + I_c \\ &= 1384.31\underline{/-75°} + 1337.14\underline{/90°} \\ &= 358.29\underline{/0°} \text{ A} \end{aligned}$$

The voltage across the motor is

$$V\underline{/0°} = \frac{480}{\sqrt{3}}\underline{/\theta_2} - 358.29\underline{/0°}\,(0.018\underline{/90°})$$

from which

$$\theta_2 = 1.4°$$

*The cost of this capacitor is about 40% of the cost of the motor.

and

$$V = 277.05 \text{ V/phase}$$
$$= 479.86 \text{ V, L-L}$$

The torque developed at starting is

$$T = 2.2\left(\frac{479.86}{480}\right)^2 = \underline{\underline{2.2 \text{ pu}}}$$

The results are summarized in Table 3-4.

TABLE 3-4 Summary of the results of Example 3-4

Operating Condition	*Starting Torque (pu)*	*Feeder Current at Starting (A)*
Nominal	2.2	1384.31
Without the capacitor	1.82	1384.31
With the capacitor	2.2	358.29

Exercise 3-9

For the motor whose characteristics as a function of speed are shown in Fig. 3-23, the nominal speed is 1765 r/min. Estimate the current, power factor, and torque of the motor when its speed varies by ±2% of its nominal value. Tabulate the results.

3.3 Measurement of Equivalent-Circuit Parameters

The parameters of the approximate equivalent circuit* of a three-phase induction machine can be obtained from the equivalent standard transformer tests. The actual measurements are carried out on the stator of the machine. Consequently, the calculated parameters and the equivalent circuit are as seen from the stator.

*For a more accurate and complete discussion of tests on induction machines, see IEEE Test Code for Polyphase Induction Motors and Generators No. 112A (New York: Institute of Electrical and Electronic Engineers, 1964).

Measurement of Stator Resistance

Per-phase stator resistance (R_1) is measured by any of the standard dc methods. This resistance is usually adjusted in order to compensate for the effects of higher operating motor temperatures. Usually $R_{ac} \approx 1.3\ R_{dc}$.

Locked-Rotor Test ($s = 1$)

The locked-rotor test on an induction machine—often referred to as the blocked-rotor test—corresponds to the short-circuit test on a transformer. Keeping the rotor stationary, we find that the slip is equal to unity, and thus the equivalent mechanical load is shorted.

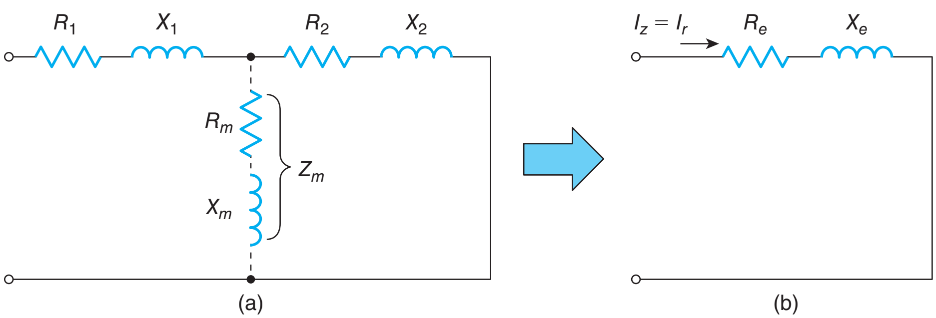

FIG. 3-25 Equivalent circuit corresponding to locked-rotor test ($s = 1$): **(a)** actual (Z_m large); **(b)** approximation of (a).

Refer to Fig. 3-25(b). The locked-rotor test gives the equivalent impedance of the stator and rotor windings. That is,

$$Z_e = R_e + jX_e \tag{3.57}$$

These parameters are calculated by measuring the stator's 3-ϕ power (P_z), the line voltage (V_z), and the line current (I_z).

From basic considerations, we have

$$Z_e = \frac{V_z}{I_z} \tag{3.58}$$

$$R_e = \frac{P_z}{I_z^2} = R_1 + R_2 \tag{3.59}$$

and

$$X_e = \sqrt{Z_e^2 - R_e^2} = X_1 + X_2 \tag{3.60}$$

where V_z and P_z are the phase-to-neutral voltage and per-phase power, respectively.

The empirical relationships between the stator and rotor reactances are given in Table 3-5.

TABLE 3-5 Empirical relationships for induction motor leakage reactances (from IEEE Test Code)

Type of Motor	*Class A*	*Class B*	*Class C*	*Class D*	*Wound-Rotor*
X_1	$0.5X_2$	$0.67X_2$	$0.43X_2$	$0.5X_2$	$0.5X_2$

Based on data from IEEE Test Code

The current for the locked-rotor test (I_z) is usually the rated current of the motor:

$$I_z = I_{\text{rated}}$$

The voltage (V_z) required by this test is normally a small percentage of the rated voltage. Usually,

$$V_z \approx (10\% \rightarrow 25\%)V_{\text{rated}}$$

The power measured is essentially the rate of heat loss (I^2R) in the stator and rotor windings under normal operating conditions. A small percentage of P_z is core loss, which is normally assumed to be negligible.

For class B and C 60 Hz motors, the frequency recommended for the locked-rotor test is 15 Hz. This frequency is necessary in order to simulate a rotor-heating effect equivalent to running conditions. The reactances measured at 15 Hz are only one-quarter of their actual values.

For class A motors, the recommended frequency is the same as that required under normal operating conditions.

No-Load Test ($s = 0$)

The no-load test of an induction motor corresponds to the open-circuit test on a transformer. The machine runs unloaded at a speed very close to synchronous, while the excitation power (P_{exc}), the voltage (V_{exc}), and the current (I_{exc}) are measured.

The excitation voltage should be equal to the rated voltage of the motor so that rated flux conditions prevail. That is,

$$V_{\text{exc}} = V_{\text{rated}}$$

The excitation current is a large percentage of the rated current:

$$I_{\text{exc}} = (25\% \rightarrow 40\%)I_{\text{rated}}$$

The excitation current is larger than that of a transformer with the same kVA and voltage ratings because of the larger air gap between the rotor and stator structures and because of the rotational losses that accompany the no-load test.

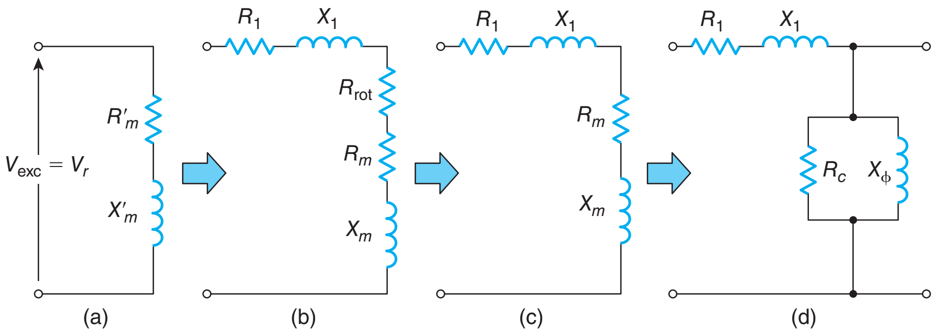

FIG. 3-26 Equivalent circuit obtained from the no-load test: **(a)** measured parameters ($Z'_m = R'_m + jX'_m$); **(b)** detailed representation of the measured parameters, $Z'_m = (R_1 + R_{rot} + R_m) + j(X_1 + X_m)$; **(c)** the resistance (R_{rot}) that represents the rotational losses; **(d)** parallel representation of the magnetizing impedance shown in (c).

The data of the no-load test gives the so-called gross open-circuit motor impedance (Z'_m), as shown in Fig. 3-26(a).

$$Z'_m = R'_m + jX'_m \tag{3.61}$$

This impedance includes the stator and magnetization impedances as well as the rotational losses in their equivalent ohmic values. From basic definitions, we have

$$Z'_m = \frac{V_{exc}}{I_{exc}} \tag{3.62}$$

$$R'_m = \frac{P_{exc}}{I^2_{exc}} \tag{3.63}$$

and

$$X'_m = \sqrt{Z'^2_m - R'^2_m} \tag{3.64}$$

The no-load resistance (R'_m) is equal to the stator resistance (R_1) plus the equivalent resistances of the core loss (R_m) and the rotational losses. (See Fig. 3-26(b).)

$$R'_m = R_1 + R_m + R_{\text{rotational losses}} \tag{3.65}$$

The rotational losses* are normally incorporated in the mechanical load and are considered part of the power consumed by the equivalent load resistance $R_2(1 - s)/s$.

*The rotational losses are made up of the stray-load losses and the mechanical losses. The mechanical losses can be accurately measured by plotting, at no-load, the power versus voltage squared. The intercept with zero voltage gives the mechanical losses.

Reasonable results are obtained by assuming the actual core-loss resistance to be two-thirds of the total no-load resistance. That is,

$$R_m = \frac{2}{3} R'_m \text{ ohms/phase} \quad \textbf{(3.66)}$$

The no-load reactance (X'_m), as shown in Fig. 3-26(c), is equal to the sum of the magnetizing and stator reactances. That is,

$$X'_m = X_1 + R_m \quad \textbf{(3.67)}$$

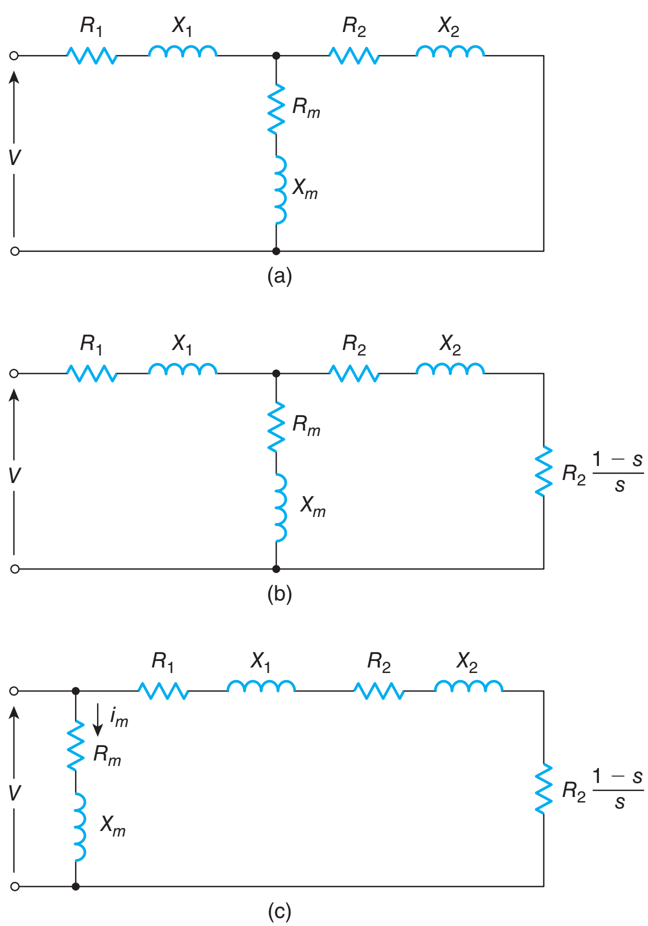

FIG. 3-27 Per-phase approximate equivalent circuits as seen from the stator: **(a)** rotor stationary, **(b)** rotor running, **(c)** a further approximation of (b); the magnetizing impedance has been relocated in order to simplify the calculations.

The stator reactance is relatively small, and for all practical purposes,

$$X'_m \approx X_m \tag{3.68}$$

The series magnetizing impedance is often represented by an equivalent parallel impedance, as shown in Fig. 3-26(d).

The parallel representation of the magnetizing impedance is more common and can easily be found from its equivalent series representation, as discussed in Chapter 2 (Eqs. (2.24) and (2.25)).

The parameters of the motor's equivalent circuit are shown in Fig. 3-27. The most commonly used approximate equivalent circuit for the engineering analysis of induction machines is that of Fig. 3-27(c).

Example 3-5

A 100 kW, three-phase, 60 Hz, 480 V, star-connected, 1146 r/min, Class C induction motor yielded the following results when tested:

a. Average value of dc resistance between stator terminals: 0.20 ohms

b. No-load test:

$$V_{exc} = 480 \text{ V, L-L}$$

$$P_{exc} = 1920 \text{ W, three-phase}$$

$$I_{exc} = 40 \text{ A}$$

c. Locked-rotor test:

$$V_z = 32 \text{ V, L-L}$$
$$P_z = 2560 \text{ W, three-phase}$$
$$I_z = 65.6 \text{ A}$$
$$f = 15 \text{ Hz}$$

Determine the per-phase equivalent circuit of the motor.

SOLUTION

No-Load Test (Fig. 3-28)

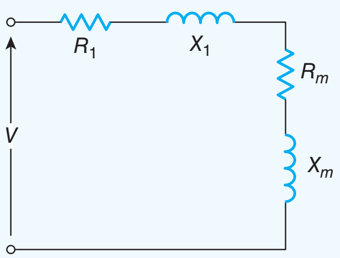

FIG. 3-28 Circuit corresponding to no-load test.

Using Eq. (3.63),

$$R'_m = \frac{P_{\text{exc}}}{I^2_{\text{exc}}} = \frac{1920}{3(40)^2} = 0.40\ \Omega/\text{phase}$$

From Eq. (3.66),

$$R_m = \frac{2}{3}(0.40) = 0.27\ \Omega/\text{phase}$$

The per-phase stator resistance is

$$R_1 = \frac{0.2}{2} = 0.10\ \Omega/\text{phase}$$

The rotational loss equivalent resistance is found by using Eq. (3.65).

$$R_{r.l.} = 0.40 - 0.10 - 0.27 = 0.033\ \Omega/\text{phase}$$

This resistance is not explicitly shown in the equivalent circuit, but its power consumption will be part of the power developed by the motor.

From Eq. (3.62), the gross magnetizing impedance is

$$Z'_m = \frac{V_{\text{exc}}}{I_{\text{exc}}} = \frac{480/\sqrt{3}}{40} = 6.92\ \Omega/\text{phase}$$

The gross magnetizing reactance is

$$X'_m = \sqrt{6.92^2 - 0.40^2} = 6.92\ \Omega/\text{phase}$$

Locked-Rotor Test (Fig 3-29)

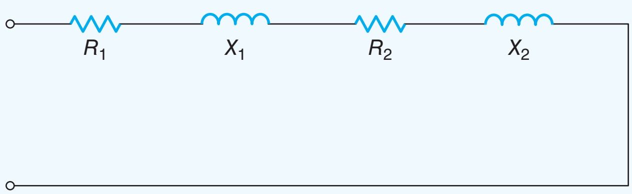

FIG. 3-29 Locked-rotor test.

From the data of the locked-rotor test and Eqs. (3.58) and (3.59), we obtain

$$Z_e = \frac{V_z}{I_z} = \frac{32/\sqrt{3}}{65.6} = 0.28\ \Omega/\text{phase}$$

$$R_e = \frac{P_z}{I_z^2} = \frac{2560}{3(65.6)^2} = 0.2\ \Omega/\text{phase}$$

The rotor resistance, as seen from the stator, is found by using Eq. (3.59).

$$
\begin{aligned}
R_2 &= R_e - R_1 \\
&= 0.20 - 0.10 \\
&= 0.10\ \Omega/\text{phase}
\end{aligned}
$$

The equivalent motor reactance is found by using Eq. (3.60).

$$X_e = \sqrt{Z_e^2 - R_e^2} = \sqrt{0.28^2 - 0.20^2} = 0.20\ \Omega/\text{phase}$$

The stator and rotor reactances can be found by using Tables 3-5 and Eq. (3.60), as follows:

$$X_e = X_1 + X_2$$

Substituting, we get

$$0.2 = X_1 + \frac{X_1}{0.43}$$

From the last two equations,

$$X_1 = 0.06\ \Omega/\text{phase}$$

and

$$X_2 = 0.14\ \Omega/\text{phase}$$

The above reactances were measured at a frequency of 15 Hz. Thus, at the rated frequency of 60 Hz,

$$X_1 = 4(0.06) = 0.24\ \Omega/\text{phase}$$

and

$$X_2 = 4(0.14) = 0.56\ \Omega/\text{phase}$$

The magnetizing reactance is found by using Eq. (3.67):

$$
\begin{aligned}
X_m &= X'_m - X_1 \\
&= 6.93 - 0.24 = 6.69\ \Omega/\text{phase}
\end{aligned}
$$

The per-phase equivalent circuit, as obtained from the data of the given tests, is shown in Fig. 3-30.

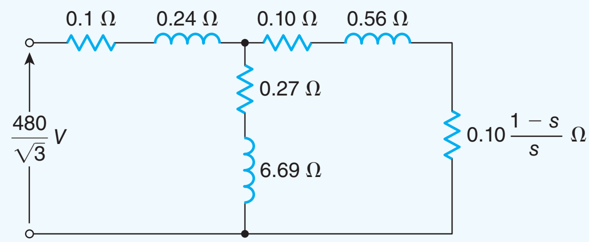

FIG. 3-30

3.4 Asynchronous Generators

When the rotor of a 3-ϕ induction machine is driven above synchronous speed and in the same direction as the synchronously rotating field, the slip—as can be seen from the general slip definition—becomes negative. The current and the voltage of the rotor windings are reversed in polarity, and the torque developed is in a direction opposite to the rotation. In this mode of operation, the machine delivers power to the stator terminals; that is, the motor acts as an induction generator.

An ordinary 3-ϕ induction motor, driven in the proper direction at hypersynchronous speeds, becomes an induction generator. The *excitation* of the generator is provided by the source of the stator voltage, and the *shaft's rotation* is derived from a water, wind, solar, steam, or gas-driven turbine. In some cases, a capacitor bank provides the excitation voltage.

Figures 3-31(a) and (b) show an elementary representation of an induction motor and an induction generator. In many respects, the induction generator is

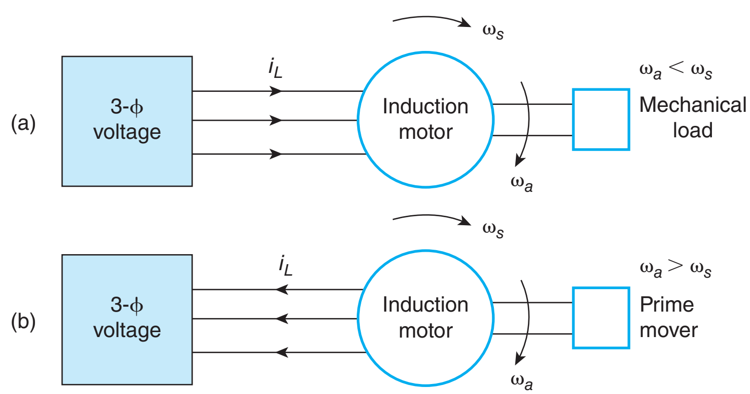

FIG. 3-31 Induction machines: **(a)** induction motor, **(b)** induction generator.

identical to the induction motor. The equivalent circuits are the same, but the energy flows in opposite directions. An induction machine that drives a hoist operates as an induction generator for descending loads and as a motor for ascending loads.

The induction generator finds increasing applications because it provides the most economical method of interconnecting a small power station to a large power-distribution network. Since the late 1970s, small power-generating stations have been operating in many places because the law requires utilities to purchase, at reasonable rates, the electricity produced by their own consumers.

The induction generator is not *self-excited*; it requires magnetizing kVAR (kilovolt-amperes reactive), which is drawn from the power-distribution network to which the generator is connected. If they are the right size, capacitor banks connected in parallel to the generator can compensate for the magnetizing or reactive power drawn by the induction generator. However, the power factor of the network—as seen from the terminals of the generator—depends on the impedance of the other loads that are connected to the same power grid. The magnetization of the machine can also occur through:

- The residual magnetism of the machine's metallic structure.
- A permanent magnet in the rotor.

The prime movers of induction generators are equipped with speed-control governors that limit the no-load speed of the machines to safe levels. This safeguard is necessary because, when the electrical load of the generator is suddenly disconnected, its rotor—owing to the constant prime mover's power and diminished electrical counter-torque—will overspeed.

Overspeeds are undesirable because the resulting centripetal and centrifugal forces may damage the rotor if the rotor is not designed to accommodate them. The level of overspeed depends on the ability of the control apparatus to instantly reduce the power developed by the prime mover. It is not always possible to instantly reduce the power, however. If a hydropower station were to attempt this operation, for example, destructive pressure rises would be produced within upstream water conduits (penstocks).

High overspeeds are also undesirable because they may induce high stator voltages. The magnitude of the induced voltage depends on the machine's electrical time constant (L/R) and on the prime mover's speed. For example, if an induction generator has an open-circuit time constant of 6 seconds, then its terminal voltage after 2 seconds—at a constant prime-mover speed—will be 72% ($e^{-2/6}$) of nominal voltage.

If, however, the speed of the prime mover is increased at the same time by 200%, then the terminal stator voltage will be about 144%. This overvoltage has undesirable effects, not only on the generator but also on the auxiliary equipment (potential transformers, relays, etc.) that is part of the generator's control apparatus.

The power rating of installed induction generators is usually in the range of 20 kW to 50 MW. If a generator's voltage rating is different from that of the distribution network, a transformer is used to interconnect the two voltage levels.

Equivalent Circuit

The per-phase equivalent circuit of an asynchronous generator is shown in Fig. 3-32. The generated voltage (V_g) is directly proportional to the machine's magnetic field (ϕ) and the rotor's speed (ω). That is,

$$V_g = K\phi\omega$$

The terminal voltage (V_t) and frequency are slightly higher than those of the utility's transmission lines. The stator (Z_1), rotor (Z_2), and magnetizing impedances (Z_m) are slightly higher than those corresponding to motor operation because the frequency of the generated voltage is larger than that when the machine operates as a motor.

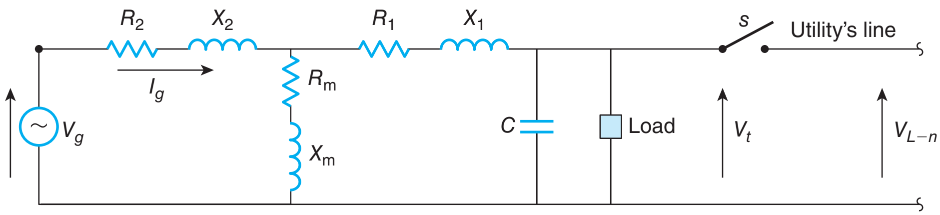

FIG. 3-32 Per-phase equivalent circuit of an asynchronous generator.

EXAMPLE 3-6

If a 100 kW, 480 V windmill asynchronous generator operates at 1225 rpm and delivers 115 A at 0.9 Pf to a three-phase 60 Hz system, estimate:

a. The generator's operating slip.

b. The maximum value of the capacitor in order to provide the machine's magnetizing field.

c. The ratio of the induction generator/induction motor reactances.

d. What should be the line-to-line voltage of the utility's voltage for proper interconnection to the asynchronous generators?

SOLUTION

a. The closest synchronous speed of the rotating field is

$$\eta_s = 120\frac{f}{P} = 120\frac{(60)}{P}$$

For a six-pole machine,

$$\eta_s = 120\frac{(60)}{6} = 1200 \text{ rpm}$$

and the machine's corresponding slip is

$$s = \frac{1200 - 1225}{1200} = \underline{\underline{-0.02 \text{ pu}}}$$

b. The apparent power of the machine is

$$|S| = \sqrt{3}(480)(115)$$
$$= 95.61 \text{ kVA}$$

and the nominal reactive power (Q) of the machine is

$$\begin{aligned} Q &= P \tan \theta \\ &= 95.61(0.9) \tan(\arccos 0.9) \\ &= 41.68 \text{ kVAR} \end{aligned}$$

From which

$$Q = VI \sin \theta_c = VI \sin 90^\circ = V\frac{V}{Z} = V^2 \omega C$$

and

$$C = \frac{41.68 \times 10^3}{(480)^2(377)} = \underline{\underline{480 \ \mu\text{F}}}$$

c. The motor's nominal operating slip is

$$s = 0.02 \text{ pu}$$

and the ratio of the reactances is

$$\begin{aligned} \frac{X_g}{X_m} &= \frac{2\pi fg(L)}{2\pi fm\ (L)} \\ &= \frac{f_g}{f_m} = \frac{f\,(1.02)}{f\,(0.98)} = \underline{\underline{1.04}} \end{aligned}$$

d. The terminal voltage and frequency of the asynchronous generator must be slightly larger than those of the utility's interconnecting systems.

Exercise 3-10

Briefly elaborate on the following:

a. On interconnecting an asynchronous generator to a utility's network, the utility's terminal voltage must be slightly smaller than that of the asynchronous generator.
b. The impedances of the induction motor are smaller than those of the asynchronous generator.
c. How can the phase sequence of the voltages be changed?
d. Under what circumstances can a capacitor furnish the magnetizing field of an asynchronous generator, and what are its adverse effects?
e. The maximum kVAR of the capacitor must be equal to the motor's reactive power as registerd at the no-load test.
f. Why do the stator and rotor magnetic fields in a synchronous motor repel each other?

3.5 Controls

This section discusses the following techniques used to control the starting, stopping, and operating torque-speed characteristics of 3-ϕ induction motors.

- Reduction of the motor's high starting current and torque
- Variable frequency drives
- Coasting a motor to a stop

3.5.1 Reduction of the Motor's High Starting Current and Torque

Induction motors, upon starting at nominal line voltages, draw high starting currents and also develop high torques. The starting current could be about six times rated and the torque developed two to three times rated.

High starting currents produce high copper losses (I^2R) and may slightly increase the premises' power demand and cause voltage sags (momentary reductions in the voltage of the upstream network) that may affect the operation of equipment. In addition, the utilities may prohibit such a condition. Higher than nominal torques overstress the structure of the machine. The methods used to reduce a motor's high starting current and torque are electromechanical and electronic. In the first case, one can use in-line resistors or inductors, autotransformers, or Y-Δ starters. Electronic control includes the so-called soft start. (The soft start is discussed in Section 3.5.3.)

In-line Resistors or Inductors

The in-line resistors or inductors incorporate a start-stop push button, a contactor, and a timer that sequentially switches the in-series elements ON or OFF so that the motor's voltage and current are kept at a desirable level. The main disadvantage of such a setup is the energy dissipated upon starting.

Autotransformers

Autotransformers and in-line resistors have a similar function. The desirable voltage level of the autotransformer is selected through the secondary voltage taps of the secondary winding by using a start-stop push button, a time relay, and a contactor. The momentary change in the voltage level is, however, accompanied by high currents, which is true of all inductive circuits.

Y-Δ Starters

The Y-Δ starter incorporates simple controls through which, at starting the motor's windings, are connected in a Y-configuration; when the motor reaches about 80% of rated speed, a time delay relay switches OFF the Y-winding configuration and

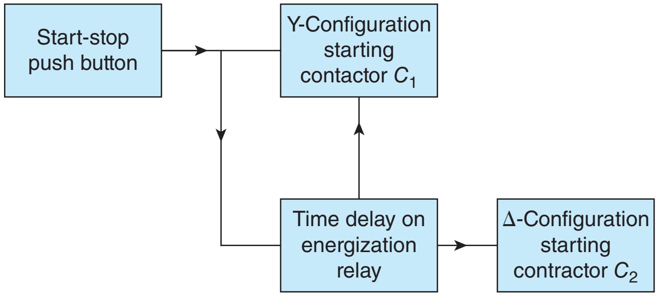

FIG. 3-33 Block diagram representation of a Y-Δ starter.

switches ON the Δ-winding configuration. This scheme reduces the motor's high starting current and torque and thus mitigates their adverse effects. The block diagram configuration for the Y-Δ starter is shown in Fig. 3-33.

Y-Winding Configuration

Refer to Fig. 3-34. From the starter to the motor's terminals, there are six wires: three for the line currents and the other three for the windings' neutral connection. Through a start-stop push button, the contactor's coil (C_1) is energized (not shown in the diagram), its contacts close (C_1), and a time delay is also energized. The motor's winding receives 0.58 of the line voltage $\frac{1}{\sqrt{3}}$, the shaft rotates, and

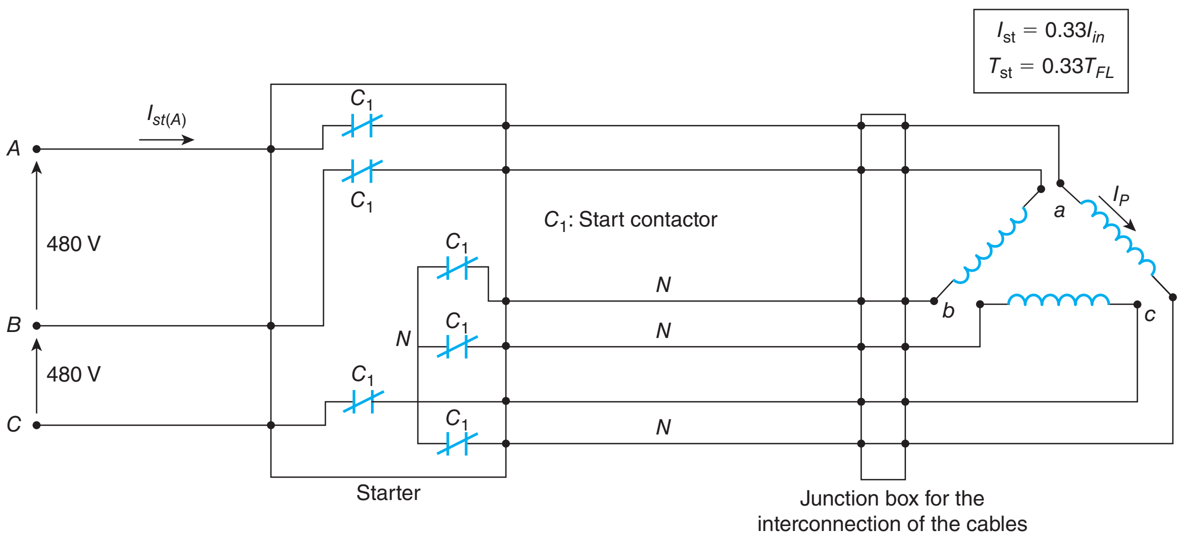

FIG. 3-34 Starting a three-phase motor. Start-connected schematic.

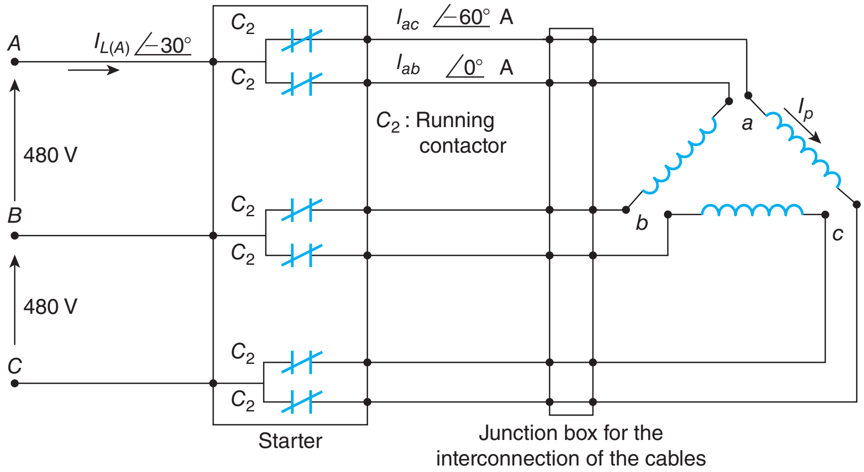

FIG. 3-35 Operating a 3-ϕ motor. Delta-connected schematic.

after a predetermined time, the coil of contactor C_1 is deenergized and the coil of contactor C_2 is energized. These steps lead to operation of the motor in the Δ-winding configuration (Fig. 3-35).

Current Considerations

At starting, the motor's windings are star-connected. The magnitude of the phase (I_P) and line currents (I_L) of a delta-connected load is

$$I_{L(\Delta)} = \sqrt{3}\, I_P$$

and

$$I_{L(\Delta)} = \sqrt{3}\left|\frac{V_{\text{L-L}}}{Z}\right|$$

where $V_{\text{L-L}}$ and Z are, respectively, the line-to-line voltage and the per-phase load impedance. When the *same* impedances are connected in a star configuration, the magnitude of the line current is

$$I_{L(\text{Y})} = \left|\frac{V_{L\text{-}L}/\sqrt{3}}{Z}\right|$$

From the last two equations, we obtain

$$I_{L(\text{Y})} = \underline{\underline{0.33 I_{L(\Delta)}}}$$

That is, when the motor's delta-connected windings at starting are connected in Y-configuration, the starting current is reduced by a factor of $\frac{1}{3}$.

Torque Considerations

The torque of a three-phase induction motor is proportional to the voltage squared. That is,

$$T \propto V^2$$

For the Y-winding and delta configuration, we obtain

$$\frac{T_{\mathrm{st\,(Y)}}}{T_{\mathrm{st\,(\Delta)}}} = \frac{\left(\dfrac{V_{L\text{-}L}}{\sqrt{3}}\right)^2}{(V_{L\text{-}L})^2}$$

from which

$$\underline{\underline{T_{\mathrm{st\,(Y)}} = 0.33\, T_{\mathrm{st\,(\Delta)}}}}$$

That is, when at starting the motor's windings are star-connected, the torque developed is also one-third that of the delta-winding configuration. In a particular application, the reduction in the torque must be verified to ensure that it is sufficient to meet the starting requirements of the load. At the instant of switching, the current and the torque of the motor increase and the windings' insulation and the motor's structure are overstressed.

Δ-Winding Configuration

Figure 3-35 presents the same motor starter for the Δ-winding configuration. Only the contacts of the contactor C_2 are shown. The line current to the starter is the nominal current of the motor. Each line current feeds two phases of the motor's windings. That is, for line "A", we have

$$I_A = I_{ab} + I_{ac}$$

$$I_A = I_{ab} \underline{/0} + I_{ac} \underline{/-60°}$$

The 60 degree phase shift is due to the standard 120 degree phase shift between voltages and currents in a three-phase system. The phase currents are of equal magnitude. That is,

$$I_P = I_{ab} = I_{ac}$$

From the last two relationships, the magnitude of the phase current is

$$I_P = \frac{I_A}{\sqrt{3}}$$

which is the standard relationship between the phase and line current in a delta-connected system. Occasionally, the problem arises regarding the size of the cables from the starter—several meters away—to the motor. These cables carry not $\frac{1}{2}$ of the line current but $\frac{1}{\sqrt{3}}$ to that of the supply line to the starter.

Example 3-7

A 200 kW, 480 V, three-phase induction motor has a starting and a nominal operating current, respectively, of 1800 A and 300 A. The motor's winding-configuration is star-connected on starting and delta-connected on steady-state operation. Determine the six-line currents from the starter to the motor at:

a. Starting.

b. Nominal operation.

SOLUTION

a. At starting, the magnitude of the line currents (I_L) is equal to the phase current (I_P)

$$I_{\mathrm{L}} = \frac{1}{3}(1800) = 600 \text{ A}, \quad \text{line “a”}$$

The currents in the lines “b” and “c” are equal to the current of line “a” and 120 degrees to it.

The neutral wires carry no current for balanced voltage supply and equal load impedances.

b. At nominal operation, the motor's winding configuration is delta, and the line currents to the starter are nominal at 300 A.

From the starter, however, to the motor's terminals,

$$\begin{aligned} I_{\mathrm{A}} &= I_{\mathrm{ab}} + I_{\mathrm{ac}} \\ &= K\underline{/0} + K\underline{/-60} \\ &= \sqrt{3}K\underline{/-30°} \end{aligned}$$

From which

$$\begin{aligned} K &= \frac{300}{\sqrt{3}}\underline{/30} \\ &= \underline{\underline{173.21\underline{/30}\ \text{A}}} \end{aligned}$$

Thus, the magnitude of the current through each of the six wires to the motor is

$$173.21 \text{ A}. \quad \text{That is, } \left(I_{\mathrm{P}} = \frac{I_{\mathrm{L}}}{\sqrt{3}}\right)$$

Briefly explain the following:

Exercise 3-11

a. The phase angle of the starting current of an induction motor is very high (about 70°) relative to that of a nominal operation (about 25°).
b. For a given number of magnetic poles, the higher the speed of the motor, the higher is its power factor.
c. A partially loaded motor operates at a low-power factor.
d. Each of the lines to Y-Δ starter supplies current to two-motor lines on a delta-winding configuration. Each of these lines carries 0.58 of the corresponding line current to the starter.
e. Draw in block diagram from the elementary control diagram of the Y-Δ motor starter.

3.5.2 Variable Frequency Drives

The variable frequency drives (VFD) refer to a control system through which the voltage and its frequency applied to an induction motor vary in such a way as to meet the torque-speed requirements of a particular mechanical load and to minimize the corresponding energy consumed. As shown in Fig. 3-36, the VFDs have four distinct sections: the rectifier, the filter, the inverter, and the controller.

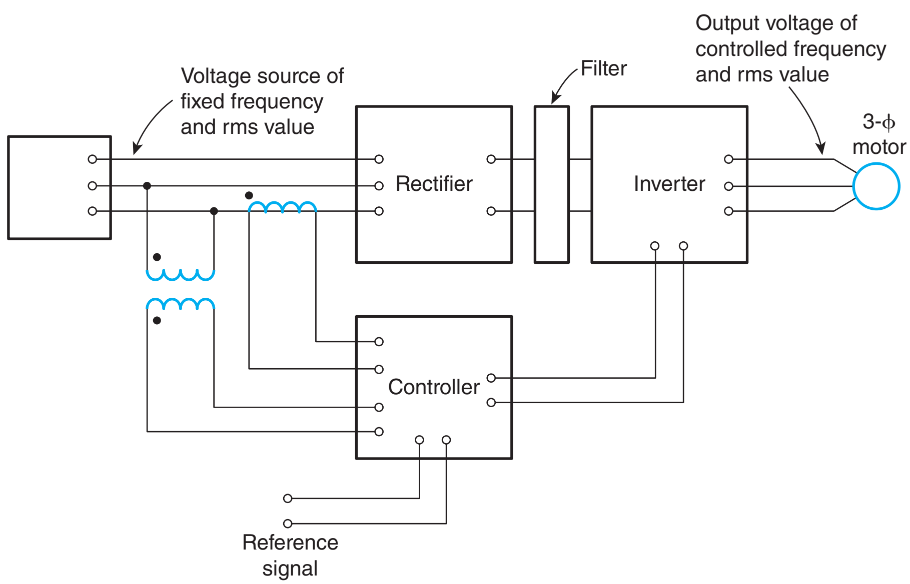

FIG. 3-36 VFD. Block diagram representation.

The rectifier is usually a six-pulse diode bridge and changes the incoming ac voltage to a dc waveform. The filter, as its name implies, blocks the transmission of high-frequency harmonics. The inverter changes the rectifier's output dc voltage waveform to a three-phase ac signal of desired voltage and frequency. The controller incorporates all the devices necessary for the protection and automatic control of the inverter's output. These components of VFD are discussed in some detail in the following sections.

Rectifiers

A rectifier receives a three-phase ac waveform at its input terminals and produces a dc voltage waveform at its output terminals. A typical six-pulse, diode-bridge rectifier is shown in Fig. 3-37(a). It is called a six-pulse rectifier because, during a complete cycle of the input voltage, its output consists of six voltage pulses. The diodes act as switches that are either open- or short-circuited, according to the relative polarity of the voltage from their anode (A) to cathode (K) (see diode D_1). When the diode's anode is at a higher potential than its cathode, the diode is shorted (forward-biased or conducting). When the diode's cathode is at higher potential than its anode, the diode is open-circuited (reverse-biased or nonconducting).

The ON-OFF condition of the diodes (such as D_1 and D_2) connected to terminals a and b depends on the magnitude of the voltage between a and $b(v_{ab})$ relative to the other line-to-line voltages. At the particular time interval when the voltage v_{ab} is the highest, then diodes D_1 and D_2 conduct. Refer to Fig. 3-37(b). The conducting period of these two diodes is from $\pi/3$ to $2\pi/3$ radians.

The conducting period of each pair of diodes can be determined in this manner. The output current pulses for a resistive load are shown in Fig. 3-37(c). According to Ohm's law, these current pulses are similar to the output voltage pulses given in Fig. 3-37(b).

Each ac supply line carries four of the six output current pulses in an alternating symmetry, thus the line currents do not have a dc component. This is shown in Fig. 3-37(d).

By definition, the average value (V_{av}) of the rectifier's output voltage is

$$V_{av} = \frac{1}{T}\int_0^T v_0 \, dt \tag{3.69}$$

where v_0 is the instantaneous value of the output voltage whose period is T seconds.

Using symmetry, and considering only the output voltage pulse that is due to v_{ab}, we have

$$V_{av} = \frac{6}{2\pi}\int_{\pi/3}^{2\pi/3} V_m \sin \omega t \, d\omega t$$

$$= -\frac{3V_m}{\pi}(\cos 120° - \cos 60°) \tag{3.70}$$

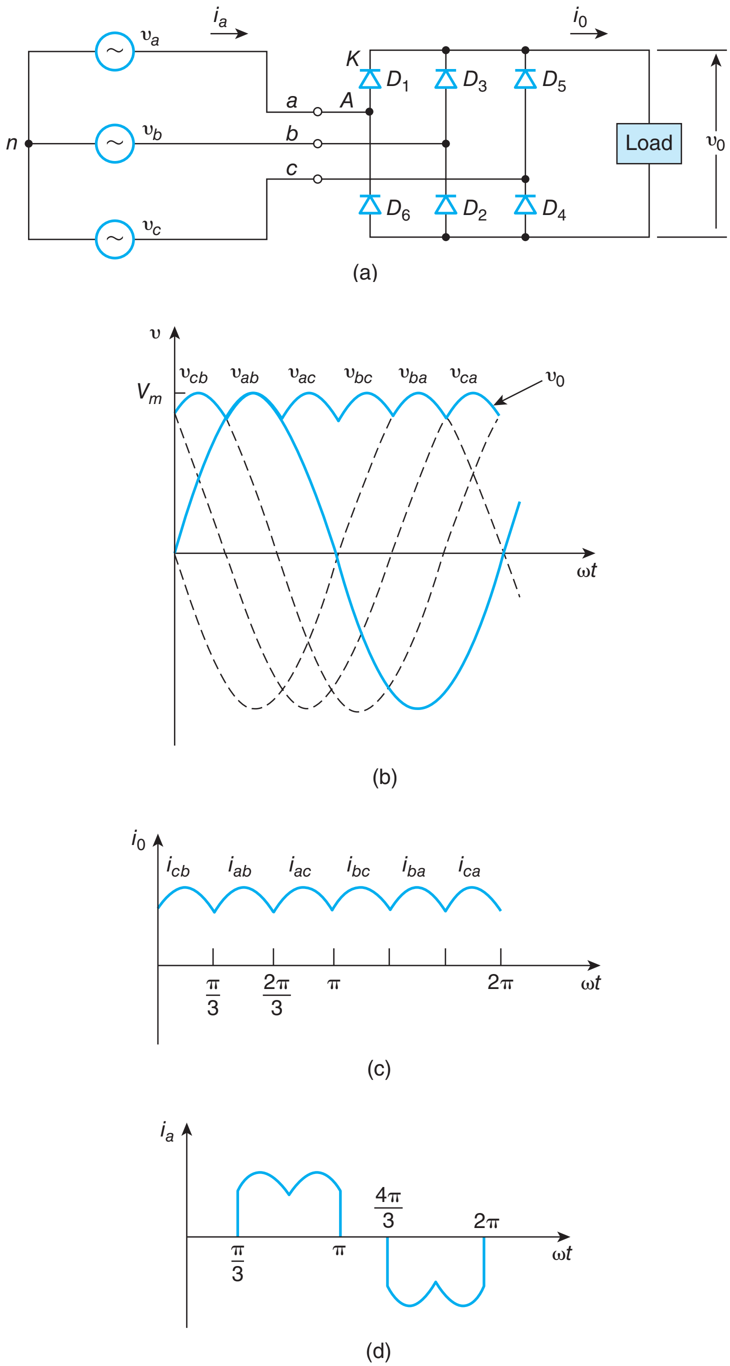

FIG. 3-37 Rectifier: **(a)** typical circuit; **(b)** input-output voltage waveforms; **(c)** output current; **(d)** line current (i_a).

from which

$$V_{av} = \frac{3}{\pi} V_m \tag{3.71}$$

or

$$V_{av} = 0.96\, V_m \tag{3.72}$$

where V_m is the maximum value of the supply line-to-line voltage.

When a capacitor of the proper size is placed across the output of the rectifier, the dc output voltage is equal to V_m. When a variable dc voltage is desirable, the diodes are replaced with control switches, such as insulated gate bipolar transistors (*I*GBTs) or thyristors. The use of *I*GBT as a switch is explained in the section on inverters later in this chapter, and the use of thyristors is described in detail in Chapter 6 (see Section 6.3).

Exercise 3-12

Assuming that the output current remains constant,

a. Derive Eq. (3.71) by considering the output voltage pulse that is due to voltage V_{cb}.
b. Prove that the rms value of the supply line current (I) is related to the dc value of the rectifier's output current (I_{av}) by the following equation:

$$I = \sqrt{\frac{2}{3}}\, I_{av}$$

Filtering Section

The harmonics reaching the motor can be minimized when a series inductor is used. The reactance of an inductor is

$$X = \omega L$$

and the higher the frequency of the harmonics, the lower the level of the harmonic components of the voltages reaching the motor. The in-parallel connected capacitor aids the function of the in-series reactor because, depending on the circuit's time constant, it will remove the ripple voltage that is always associated with the output voltage of a rectifying circuit.

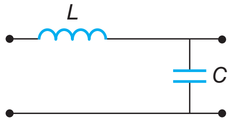

FIG. 3-38 Filtering section of a VFD.

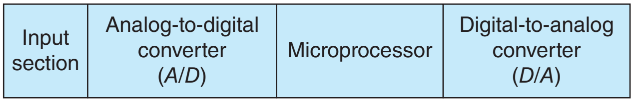

FIG. 3-39 Block-diagram representation of a digital controller.

Controller

The controller is the brain of a frequency converter. Not only does it provide a closed-loop control for the motor, but it also monitors the motor's performance and protects it from abnormal operating conditions.

A controller in block-diagram form is shown in Fig. 3-39. The input section of the regulator receives the reference signal and the outputs from the potential and the current transformers.

The output of the potential transformers (PT) is used to produce the dc voltages required to operate the various digital circuits. The output of the current transformers (CT) is directly proportional to the current drawn by the motor. When properly evaluated, it reveals the magnitudes of the motor's torque and speed, as can be seen in Fig. 3-23.

In some applications, a dc type of CT may be connected to the dc lines of the rectifier. The operation of such a CT is based on the Hall effect, not on the electromagnetic coupling of two coils. The incoming section of the controller subtracts the CT's signal from the reference signal, and the resulting signal is converted to an equivalent digital signal. This digital signal is fed into the microprocessor, which compares it to a set of stored instructions and produces an output accordingly.

The output of the microprocessor is converted to an analogue signal that controls the ON-OFF status of the *I*GBT. (See Fig. 3-40(a).)

Inverters

The function of an inverter is to produce a three-phase ac waveform of controlled frequency from a single dc voltage supply. A typical IGBT three-phase inverter is shown in Fig. 3-40(a).

Depending on the voltage from gate to emitter, each *I*GBT* in an inverter is either open-circuited or short-circuited; that is, the transistor operates as a switch and not as an amplifier. When the V_{GE} is sufficiently high, the transistor is short-circuited ($v_{CE} \approx 0$), and when the V_{GE} is zero, the transistor is open-circuited ($I_c = 0$).

When a transistor is short-circuited, it is said to be operating in the saturation region. When a transistor is open-circuited, it is said to be operating in the cut-off region.

* The *I*GBT operation is discussed on the website.

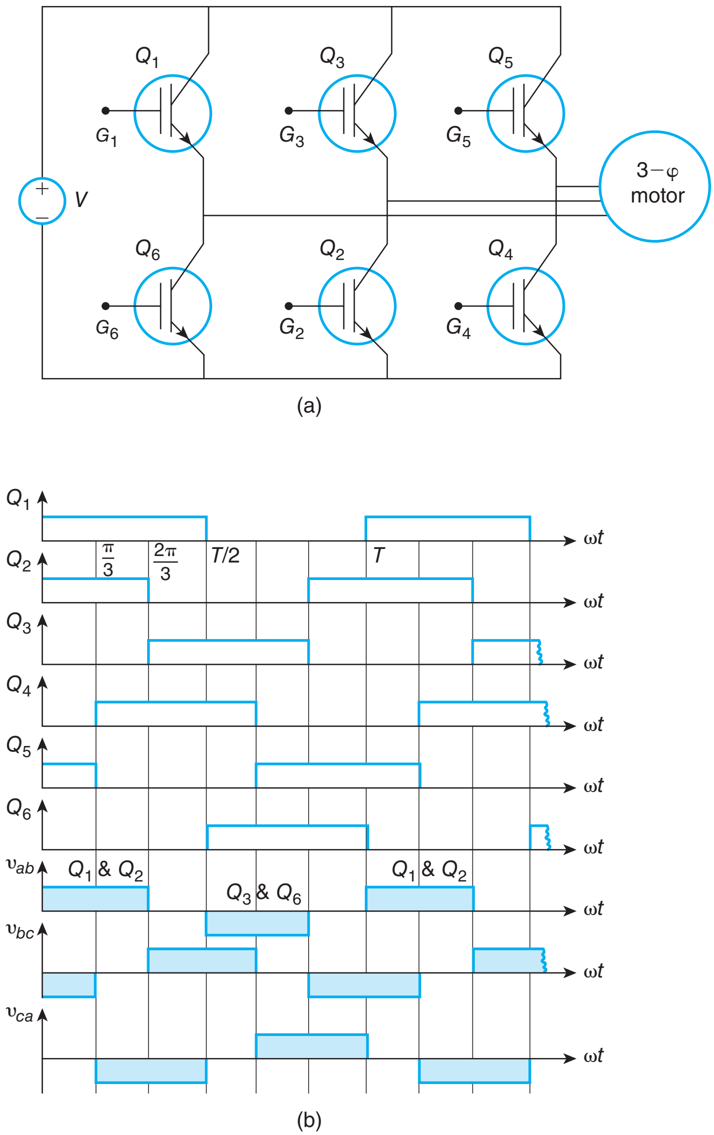

FIG. 3-40 3-ϕ Inverter: **(a)** circuit, **(b)** pertinent waveforms.

The ON and OFF status for each *I*GBT is equal to half the period ($T/2$) of the output voltage. The switching ON of a transistor is delayed $\pi/3$ radians relative to the transistor that was previously conducting. For the current to flow, two *I*GBTs must conduct simultaneously.

Consider the output voltage waveform v_{ab}. As shown in the diagram and in the following table, for each component of this output voltage, a unique pair of *I*GBTs conduct.

Time Interval of v_{ab}	*Conducting IGBTs*
$0 - 2\frac{\pi}{3}$	Q_1 and Q_2
$2\frac{\pi}{3} - \pi$	None
$\pi - 5\frac{\pi}{3}$	Q_3 and Q_4
$5\frac{\pi}{3} - 2\pi$	None

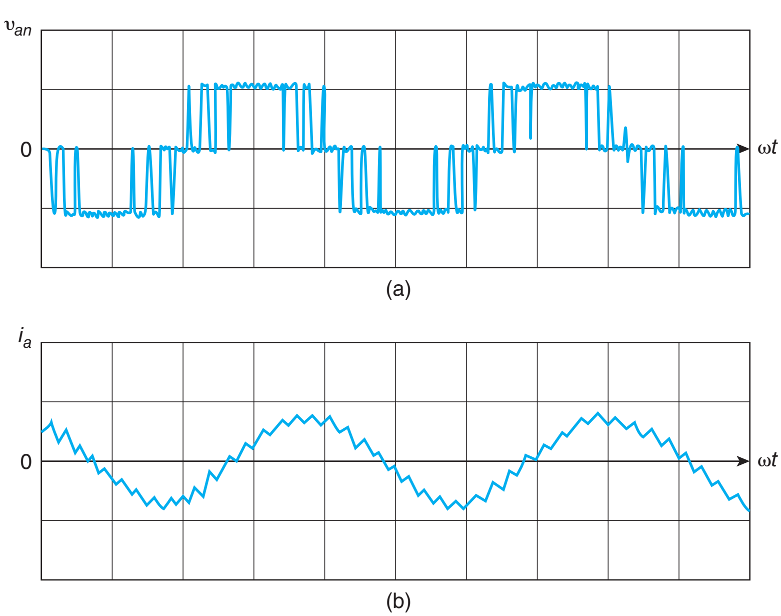

FIG. 3-41 Actual output waveforms of an inverter: **(a)** voltage, **(b)** current.

The pair of conducting transistors of the other line-to-line voltages can be determined in the same manner. The 3-ϕ voltages produced have a phase sequence ABC.

Normally, the operation of an inverter's switches produces an output voltage that, over a complete voltage cycle, is made up of a series of pulses whose width, as seen in Fig. 3-41(a), is variable. This is referred to as pulse-width modulation (PWM). Although the output voltage is made up of a series of pulses, the resulting motor current resembles a sine function, as shown in Fig. 3-41(b).

The width of the voltage pulses controls the rms value of the output voltage, and the number of pulses within a voltage cycle controls the frequency of the output voltage. The switching of the *I*GBTs produces high transient voltages due to the inductance of the load; these voltages are usually bypassed by connecting a diode across the collector-emitter terminals.

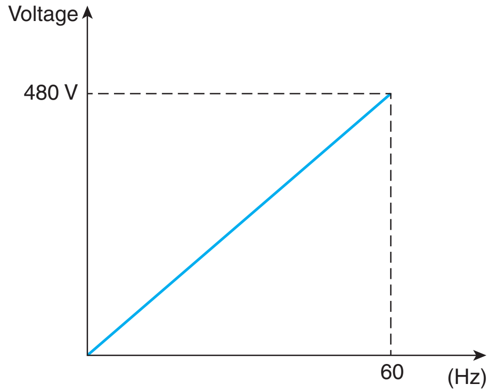

FIG. 3-42 The inverter's output voltage-frequency variation for constant machine magnetization.

The output voltage-frequency characteristic of a frequency converter is shown in Fig. 3-42. At frequencies above 60 Hz, the voltage is kept constant at the motor's rated voltage. At operating frequencies between 5 Hz and 60 Hz, the voltage is proportional to frequency.

In order to maintain a motor's nominal magnetization, the voltage/frequency ratio must be kept constant. That is, for a 480 V, 3-ϕ, 60 Hz motor, the voltage to frequency ratio is

$$\frac{480}{60} = 8.0$$

(See Eq. (1.148))

For load conditions that require higher than nominal speed, the frequency is increased, the voltage remains constant, and for constant output power the torque is inversely proportional to the speed.

Example 3-8

The speed, power, and torque of a 460 V, 60 Hz, 3-ϕ induction motor are controlled through a VFD.

a. Draw the voltage-frequency output characteristic of the inverter.

b. When the output frequency of the inverter is reduced from 60 Hz to 30 Hz, what is the rms value of the inverter's output voltage?

c. What is the reason that up to nominal speed, the voltage/frequency ratio should be kept constant?

d. Explain why, as a result of increasing the inverter's output above 60 Hz while the power to the motor is kept constant, the motor's torque is inversely proportional to the speed and the motor's magnetization is reduced.

e. Why, for a constant load torque, is the motor's power proportional to the frequency?

SOLUTION

a. See Fig. 3-42.

b. The voltage to frequency ratio is

$$\frac{V}{f} = \frac{460}{60} = 7.67$$

Thus,

$$V = 7.67f$$
$$V = 7.67(30)$$
$$V = 230\text{ V, line-to-line}$$

(The motor's magnetization did not change.)

c. The voltage to frequency ratio is maintained at a constant in order to operate the motor at a nominal magnetization level.

d. I) $P = T\omega$ and $T = \dfrac{P}{\omega}$

II) From Eq. (1.148), by increasing the frequency, the level of magnetic field is reduced.

e. $P = T\omega = TK_1 f = K_2 f$

Example 3-9

A 10 HP, 480 V, 1770 rpm, 3-ϕ, induction motor drives a ventilator that is equipped with inlet damper control. The ventilator at the operating region has the pressure (h_2) versus air flow (Q_2) characteristic given by

$$h_2 = 2\,Q_2^2$$

and at nominal operating condition

$$h_1 = (1)\,Q_1^2$$

The fan is to operate for six months per year continuously at 80% of rated air flow. Determine the energy consumed by using:

a. Damper control

b. VFD

(**Note:** The pressure versus flow characteristic is derived by the process or mechanical engineers).

SOLUTION

a. The system parameters designated with subscripts 1 and 2 are

$$h_2 = 2\,Q_2^2$$
$$h_1 = (1)\,Q_1^2$$

and

$$h_2 = 2\left(\frac{Q_2}{Q_1}\right)^2$$
$$= 2\left(\frac{0.8}{1.0}\right)^2$$
$$= 1.28$$

The air power is equal to pressure times rate of air flow.

$$HP_2 = h_2\,Q_2 \quad \text{or} \quad HP_2 = HP_1\left(\frac{h_2}{\mathrm{h}_1}\left(\frac{Q_2}{Q_1}\right)\right)$$

Substituting for the ratio of the pressures, we obtain

$$HP_2 = HP_1\frac{2\,Q_2^3}{Q_1^3} = (HP_1)\,2\left(\frac{0.8}{1}\right)^3 = \underline{\underline{1.024HP_1}}$$

Corresponding energy is

$$W_1 = 6 \times 30 \times 24(1.024 \times 0.746) = \underline{\underline{3300\ \text{kWh}}}$$

b. Based on the fan laws commonly known as affinity laws, we have

$$P \propto Q^3$$

Thus,

$$\frac{P_2}{P_1} = \left(\frac{Q_2}{Q_1}\right)^3$$
$$P_2 = P_1\left(\frac{0.8}{1}\right)^3$$
$$= 0.512P_1$$
$$= 0.512(10)(0.746)$$
$$= 3.82\ \text{kW}$$

and energy (W_2):

$$W_2 = 6 \times 30 \times 24(3.82) = \underline{\underline{16{,}502 \text{ kWh}}}$$

For comparison purposes, see the following table:

No.	Method of Rate of Air Flow Control	Energy Consumption in kWh
1	Use of dampers	16,502
2	VFD	3300

3.5.3 Soft Start

Soft start refers to the gradual increase in the motor's speed by reducing the time interval of the voltage applied to the motor. This increase is accompanied by a reduction in the motor's starting current and torque. Although the peak value of the applied voltage does not change, its rms value is reduced accordingly. A soft start control circuit could be part of a VFD or a much simpler circuit incorporating *I*GBTs and a timer.

Reduction of the voltage's rms value by using 2-*I*GBT per phase is illustrated in Fig. 3-43. For simplicity, only one phase is shown. One *I*GBT controls the duration

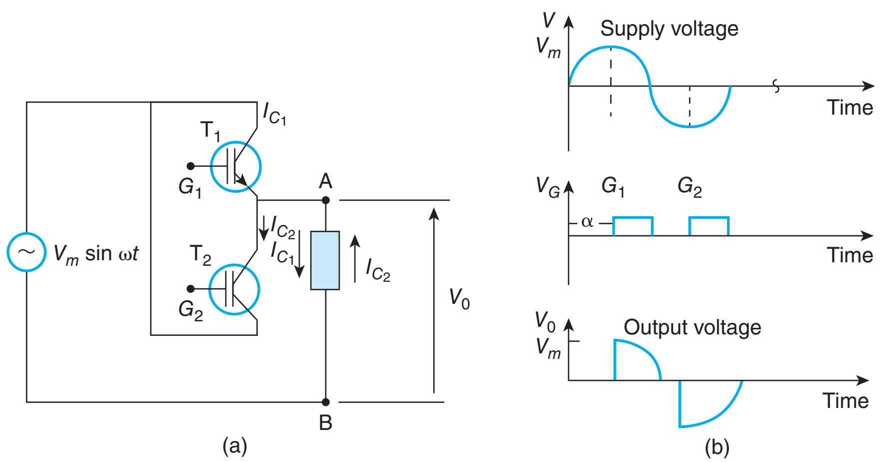

FIG. 3-43 *I*GBT control of starting current: **(a)** Circuit. **(b)** Pertinent waveforms.

of the time interval of the positive part of the voltage cycle, and the other controls that of the negative cycle. The starting point of the voltage cycle depends on the application of the gating pulse, which is commonly known as the firing angle (α).

In Fig. 3-43(a), when the supply voltage is positive and T-1 is triggered, current flows to the load from terminal A to B. When the supply voltage is negative and T-2 is triggered, current flows to the load from terminal B to A.

In Fig. 3-43(b), the firing angles for T-1 and T-2 are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$ radians, respectively. The voltage to the load (V_0) is as shown.

Example 3-10

The rms value of the current through a 208 V, 60 Hz, 3-ϕ resistive load is controlled through one *I*GBT per phase. The *I*GBT's firing angles for phase "a" are $\frac{\pi}{3}$ and $\frac{4\pi}{3}$ radians. Estimate the rms load voltage.

SOLUTION

$$V_0^2 = \frac{2}{2\pi}\int_{\pi/3}^{\pi}(V_m \sin \omega t)^2\, d\omega t$$

$$= \frac{V_m^2}{2\pi}\int_{\pi/3}^{\pi}(1 - \cos 2\omega t)\, d\omega t$$

$$= \frac{V_m^2}{2\pi}\left[\omega t\Big|_{\pi/3}^{\pi} - \frac{1}{2}\sin 2\omega t\Big|_{\pi/3}^{\pi}\right]$$

$$= \frac{V_m^2}{2\pi}\left[\frac{2\pi}{3} - \frac{1}{2}(\sin 360 - \sin 120)\right]$$

$$= 0.40\, V_m^2 = 0.4(208)\sqrt{(2)^2}$$

and

$$V_0 = \underline{\underline{186.56\, V_{\text{rms}}}}$$

Exercise 3-13

A 480 V, 60 Hz, delta-connected, three-phase motor is equipped with a soft start controller. Determine the starting current in pu when the *I*GBT firing angle is 150° and 240°.

Answer 0.12 pu

3.5.4 Plugging

A motor can be stopped by any of the following methods:

1. Removing the supply voltage.
2. Reversing the phase sequence of the applied voltages.
3. Applying a dc field to a stator phase winding while the ac voltage supply is removed.
4. Using a mechanical brake.

With each method, a specific amount of time is required to bring the motor to a complete stop.

3.6 Conclusion

The operation of induction machines is based on the development of the rotating field, on the induction principle, and on the natural tendency of magnetic fields to align their axes.

When balanced 3-ϕ currents are circulated through properly distributed coils, a rotating field is produced. This rotating field completes its loop by cutting through the rotor and stator structures. As a result, a voltage is induced in the rotor coils. This voltage, in turn, causes current to flow through the rotor windings, and thus a rotor field is produced. The stator and rotor fields try to align their magnetic axes, and as a result, the motor produces torque.

The rotor of an induction machine is of either the squirrel-cage or the wound-rotor type. Induction machines with squirrel-cage rotors are much more popular than those with wound rotors because they are less expensive to both purchase and operate. The National Electrical Manufacturers Association (NEMA) groups motors with squirrel-cage rotors into four categories—class A, B, C, or D—according to their torque-speed characteristics. Each class of motor is suited to a particular application. Class B motors, for example, are for general-purpose use and can be found in fans and blowers.

Induction machines with wound rotors are used for special applications, but because they are less efficient at higher slips, they are gradually being replaced by variable-speed drives. Wound-rotor induction machines can be found in the following special applications:

1. To operate variable torque-speed characteristics for loads that operate over a wide range of speed and torque.
2. To limit the starting current of a motor in a distribution system where the supply power system is weak (large upstream impedance).
3. To limit the starting current of motors that are required to start and restart frequently, such as those used in the crushing machines of the mining industry. The reduction in the starting current increases the number of permissible starts during any given time interval, so the motor's life is not reduced.
4. To give high torque at high slip.

A motor's slip indicates the relative motion between the speed of the synchronously rotating stator field and the speed of the shaft of the motor. The slip can be *positive* and less than unity (in an induction motor); *positive* and larger than unity (in voltage and frequency multipliers); or *negative* and less than unity (in induction generators).

For all practical purposes, the real part of the impedance of a motor as seen from its stator is inversely proportional to the slip. Thus, at starting ($s = 1$), the impedance is small, whereas under full-load conditions ($s \approx 0.015$), the impedance is very high.

Consequently, a motor's current at this operating condition—known as the rated condition—is about six times smaller than it is at starting. The large starting currents of a motor not only lower its starting-torque capability—owing to a high voltage drop in the upstream impedances—but also develop extremely high temperatures in the rotor and stator windings. These large currents may limit the number of motor starts permissible within a given time interval.

The motor's equivalent circuit is of paramount importance. It simplifies understanding of the motor's operation and facilitates analysis of a particular problem. In practice, however, the circuit parameters are often not known, and an application engineer has to make decisions based on other data (such as starting and full-load torque and currents) readily available from the manufacturers.

Induction generators are gaining in popularity because they provide a simple and the most economical method of connecting a wind turbine or a small hydropower station to a distribution network. Power stations driven by wind, solar, water, or process heat are becoming increasingly common because of recent legislation requiring utilities to buy, at reasonable rates, electricity produced by their consumers.

Variable frequency drives (VFDs) change the frequency and the voltage applied to a motor. As a result, a motor's torque-speed characteristic can be modified to meet diverse load requirements. Additionally, VFDs help reduce energy consumption per-unit output more than the other methods of speed control.

The following tables summarize the basic concepts of this chapter in their mathematical form and also furnish typical manufacturers' data for three-phase induction machines.

3.7 Tables

TABLE 3-7.1 Summary of important equations

Item	*Description*	*Remarks*
1	Magnitude of rotating field $$F_s = \frac{3}{2} F_1 \cos(\omega t - \beta)$$	Eq. (3.14)
2	Mechanical and electrical degrees or radians $$\theta = \frac{p}{2} \theta_m$$	Eq. (3.15)

TABLE 3-7.1 *(Continued)*

Item	Description	Remarks
3	Speed of rotating field in r/min $$n_s = 120\frac{f}{p}$$	Eq. (3.18)
4	Per-unit slip $$s = \frac{n_s - n_a}{n_s}$$	Eq. (3.19)
5	Voltage induced in rotor windings $$V_r = sV'_r$$	Eq. (3.23)
6	Frequency of voltage induced in the rotor windings $$f_r = sf$$	Eq. (3.24)
7	Mechanical load in equivalent electrical resistance $$R_L = R_2\frac{(1-s)}{s}$$	Eq. (3.32)
8	Torque developed $$T = \frac{V^2}{\left[\left(R_1 - \frac{R_2}{s}\right)^2 + X^2\right]\left(\frac{R_2}{\omega_s s}\right)}$$	Eq. (3.38)
9	Torque in the full-load region $$T \approx K\frac{s}{R_2}$$	Eq. (3.42)
10	Slip at maximum torque $$s_{\text{mt}} = \frac{R_2}{\sqrt{R_1^2}} + X_2$$	Eq. (3.46)
11	Motor's output torque $$T = J\frac{d\omega_a}{dt} + B\omega_a + T_L$$	Eq. (3.49)
12	Power developed $$P_d = I_2^2 R_2\frac{1-s}{s}$$	Eq. (3.56)
13	Starting current $$I_{st} = (5 \rightarrow 6)I_{fl}$$	
	Variable Frequency Drives (VFDs)	
14	Output voltage of a six-pulse bridge rectifier $$V_{av} = \frac{3}{\pi}V_m$$	Eq. (3.71)

TABLE 3-7.2 Typical Data for three-phase 60 Hz, 460 V, NEMA Design B, service factor 1.0, class insulation B, standard squirrel-cage induction motors

Power in kW	Full-Load Speed in r/min	Approximate Amperes		Starting Torque in Percent	Maxi-mum Torque in Percent	Efficiency in Percent		Power Factor in Percent	
		Full-Load	Locked-Rotor			Full-Load	50% Load	Full-Load	50% Load
0.75	1730	1.64	15	275	300	78.5	72	73	51
	870	2.3	15	135	215	70.5	61	58.5	39.5
7.5	3500	12.23	81.25	135	200	85.5	82.5	90	82
	880	15.86	81.25	125	200	83	79.5	71.5	52.5
20	3540	32.2	182	130	200	86.5	83	90	82.5
	885	35.2	182	135	200	88.5	87.5	80.5	64.5
40	3545	62.3	362.5	125	250	89	87	90.5	83
	880	69.3	362.5	145	230	90.5	88.5	80	65
50	3560	75.4	542.5	115	250	90.5	88	92	85
	885	86.2	542.5	145	210	91	90	80	66.5
75	3555	112.4	725	110	200	90.5	88.5	92.5	88.5
	885	128.4	725	125	200	90.5	89.5	81	70
100	3565	149.9	1085	110	240	91	88	92	88.5
	1775	151.6	1085	125	210	92	90.5	90	86

Based on data from Siemens Electric Limited

TABLE 3-7.3 Data for three-phase 60 Hz, 4000 V, totally enclosed, fan-cooled, squirrel-cage induction motors

kW	Full-Load Speed in r/min	Full-Load Amperes	Locked-Rotor Amperes	Starting Torque in Percent	Maxi-mum Torque in Percent	Efficiency in Percent		Power Factor in Percent	
						Full-Load	50% Load	Full-Load	50% Load
150	1764	28	162	100	200	91.8	88.7	84.1	71
	1178	27.1	162	120	200	91.8	88.7	87	75.5
200	589	32.4	162	60	175	90.8	88.6	73.7	53.5
	589	42.6	202	60	175	91.1	89	74.3	54.5
225	1180	40.4	242	100	175	92.3	89.2	87	76.4
	589	47.5	242	60	175	91.4	89.3	74.8	55.5
300	1182	53.8	323	60	175	92.7	89.6	87	76
400	1184	71.4	404	60	175	93	89.9	87	77.1
1500	1175	259	1510	140	210	95	94.5	88	77

Based on data from Siemens Electric Limited

TABLE 3-7.4 Typical test results and measurements of a 260 kW, 4000 V, 0.75 Pf, 0.88 efficient, 10-pole, squirrel-cage induction motor

a. Tests

Test	Frequency (Hz)	Voltage (V)	Current (A)	Power (kW)
No-load test	60	4000	33.1	9.5
Locked-rotor test	60	975	57	27
Locked-rotor test	30	537.8	57	29.2

b. Calculation of Load Characteristics at Rated Voltage and Frequency

Load in percent	25	50	75	100	125
Line current in A	35	40.2	47.9	57.6	68.7
Efficiency in percent	86.6	91.6	92.7	92.7	92.2
Power factor in percent	31.1	51.2	63.7	70.7	74.4
Slip in percent	0.32	0.65	1.01	1.4	1.83

Starting current (locked rotor) in pu: 4.6
Starting torque in pu: 1.5
Maximum torque in pu: 2.2

c. Temperature rise

Hours run: 4
Volts: 4000
Load: 100%

	Frame	Bearings	
		Load Side	Opposite Side
Temperature in °C	50	19	15

d. Winding resistance between terminals

At 20°C 1.69 ohms

e. Air-gap measurement

1.13 mm, average

Based on data from Siemens Electric Limited

TABLE 3-7.5 Typical data for three-phase, 4000 V, 60 Hz, squirrel-cage rotor induction motors

		Polar Moment of Inertia Referred to Speed of Motor in kg-m^2			Torque in Percent				Maximum Permissible Stall Time in Seconds	
Rating in kW	Number of Poles	Rotor	Load	Total	Starting	Pull-In	Maximum	Accelerating Time in Seconds	Starting From Cold	Re-Starting Immediately after Operation (Hot-Start)
200	4	6	100	106	110	90	230	20	25	20
200	4	6	40	46	110	90	230	9	25	20
250	6	31	217	248	90	70	220	20	25	20
250	14	68	34	102	135	100	260	2	20	16
300	6	46	125	171	175	130	210	8	20	16
375	6	53	150	203	175	130	210	8	20	16

Based on data from Siemens Electric Limited

3.8 Review Questions

1. What are the essential differences between squirrel-cage and wound-rotor three-phase induction machines?
2. Why are three-phase induction machines often called rotating transformers?
3. Differentiate between a synchronous speed and an actual speed.
4. What is "slip," and under what operating conditions is it:
   a. Positive and smaller than unity?
   b. Positive and larger than unity?
   c. Negative and smaller than unity?
5. What are the frequencies of rotor currents at starting and at full-load? How does frequency affect the impedance of the rotor?
6. Why is rated voltage used in the "no-load test" and near full-load current used in the locked-rotor test?
7. Why, in general, is the starting torque for a three-phase squirrel-cage induction motor 1.5 to 2 times the full-load torque, and the starting current 5 to 6 times the full-load current?
8. For the A, B, C, and D classifications of motors, how does the rotor reactance compare with that of the stator?
9. *Explain*: The torque at the starting region is inversely proportional to the operating slip, while at the full-load region, it is proportional to the slip.
10. Why is the time that it takes a motor, starting from rest, to reach its rated speed of paramount importance?

11. Describe the four methods by which a three-phase induction motor can be stopped.

12. How does the motor of a hoist change the direction of rotation?

13. What provides the excitation power for an induction generator?

14. *Explain*: The maximum rate of heat dissipation of a machine is given by its efficiency.

15. What are the advantages and disadvantages of VFDs?

3.9 Problems

3-1 A 480 V, 60 Hz, 1740 r/min, 0.90 Pf, 7.5 kW, three-phase induction motor has a full-load efficiency of 85%. Determine:

a. The full-load slip.

b. The number of poles.

c. The current drawn by the motor.

3-2 For a three-phase induction motor, show that:

a. At starting, the motor's torque is directly proportional to its rotor resistance. That is,

$$T_{st} \propto R_2$$

b. The magnitude of the maximum torque is independent of rotor resistance.

c. By neglecting stator resistance, the torque (T) at slip (s) is related to the maximum torque (T_m) and its corresponding slip (s_{mt}) by the following equation:

$$\frac{T}{T_m} = \frac{2ss_{mt}}{s_{mt}^2 + s^2}$$

3-3 A 240 V, 3-ϕ, 1140 r/min, three-phase induction motor has the per-phase equivalent circuit shown in Fig. P3-3.

a. Neglecting the rotational losses and the magnetizing impedance, determine:

1. The synchronous speed of the stator field.

2. The per-unit slip.

3. The line current at starting.

4. The full-load current, power factor, torque, and efficiency.

5. The per-unit impedance of the stator and rotor circuits.

6. The slip and current at maximum torque.

7. The maximum torque.

b. Repeat (a3), taking the magnetizing impedance into consideration.

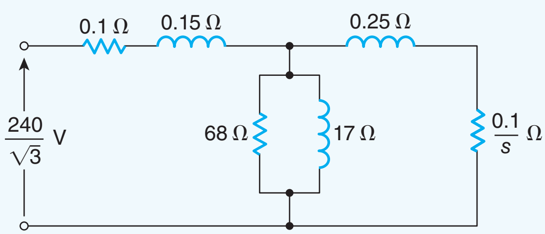

FIG. P3-3

3-4 A 480 V, six-pole, 1140 r/min, three-phase induction motor has the per-phase equivalent circuit shown in Fig. P3-3. Neglecting stator resistance, determine:

a. The full-load slip.

b. The slip at maximum torque.

c. The starting torque in pu.

d. The maximum torque in pu.

3-5 A 440 V, 60 Hz, 1746 r/min, 60 kW, star-connected, class B, three-phase induction motor yielded the following results when tested:

1. Average value of dc resistance between stator terminals = 0.16 Ω.

2. Locked-rotor test:

$Vz = 28$ V, line-to-neutral

$Pz = 2.24$ kW, three-phase

$I_z = 60$ A

3. No-load test:

$V_{exc} = 440$ V, line-to-line

$P_{exc} = 1.6$ kW, three-phase

$I_{exc} = 12$ A

Determine:

a. The per-phase approximate equivalent circuit.

b. The torque developed at full-load.

3-6 Figures P3-6(a) and (b) show, respectively, the one-line diagram and the per-phase equivalent circuit of a 3-ϕ wound-rotor induction motor. The motor drives a load whose torque-speed characteristic is given by

$$T_L = 50 + 4 \times 10^{-2} n_a \text{ N} \cdot \text{m}$$

The actual speed (n_a) is expressed in r/min. Neglecting the effects of the magnetizing current, determine:

a. The load viscous damping factor (B) in N · m/rad/s.

b. The minimum required additional rotor resistance that must be inserted in each phase in order to start the motor.

c. The operating speed with the external resistance in the rotor circuit reduced to zero.

3-7 A ventilating fan whose torque is proportional to the square of its speed is driven by a squirrel-cage induction motor at nominal conditions.

a. Assuming that the torque requirements of the fan change by ± 10% in comparison to the full-load torque, estimate qualitatively how this will affect (increase or decrease) the speed, efficiency, line current, and power factor of the motor.

b. Repeat (a), assuming that the motor is oversized by 10% above the nominal requirements.

c. Repeat (a), assuming that the motor is undersized by 10% below the nominal requirements.

3-8 The 208 V, 60 Hz, three-phase, four-wire distribution system shown in Fig. P3-8 delivers power to a three-phase induction motor and to single-phase loads. The starting current of the three-phase

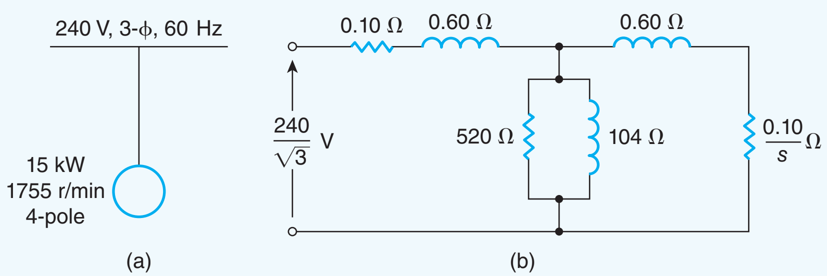

FIG. P3-6

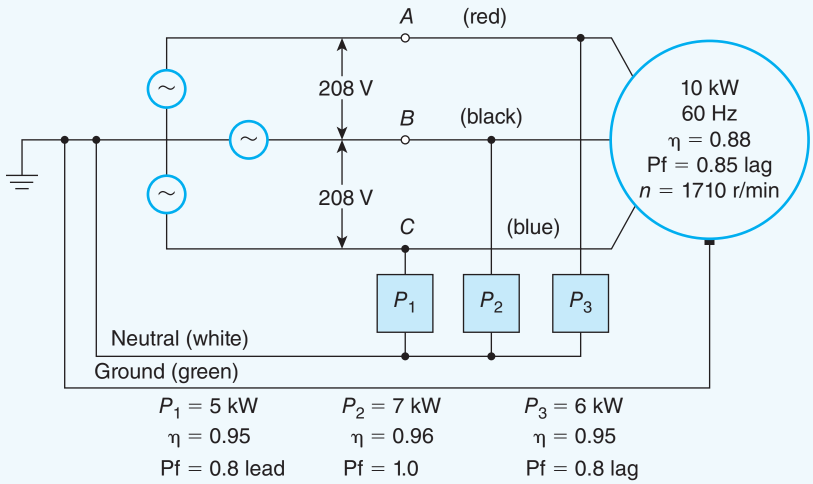

FIG. P3-8

induction motor is four times the full-load value and lags the applied voltage by 84°. Determine the currents through the transmission line and through the neutral wire of the distribution system for each of the following operating conditions:

a. The motor at starting.

b. The motor running at rated condition.

3-9 Referring to Table 3.7.4(a):

a. Determine the maximum permissible size of a capacitor bank that can be used to correct the power factor of a 260 kW motor. Assume that the motor and capacitor bank are switched through the same disconnect device.

b. What is the power factor of the motor and capacitor, as seen from their common disconnect device?

3-10 The three-phase induction motor shown in Fig. P3-10 has a starting torque and current as follows:

$$T_{st} = 1.8 \text{ pu}, \qquad I_{st} = 6\angle{-70^\circ} \text{ pu}$$

The reactances of the cables are as shown. When the 500 kW load draws rated current and the 200 kW motor starts, determine:

a. The torque developed by the motor.

b. The size of the capacitor bank that, when connected in parallel with the motor, will minimize the starting current through the upstream network.

c. The torque developed at minimum starting current.

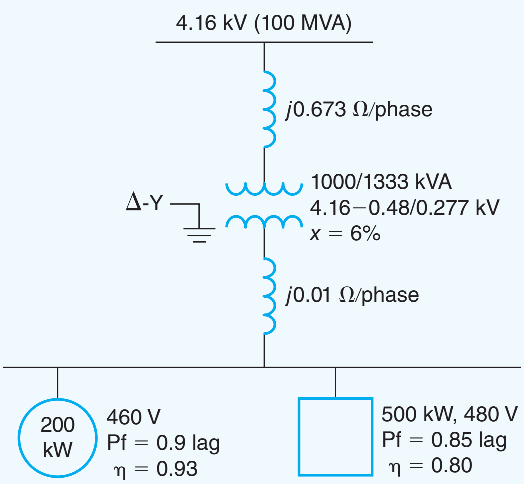

FIG. P3-10

3-11 A 480 V, 3-ϕ, 60 Hz, six-pole induction motor has the per-phase equivalent circuit shown in Fig. P3-11. The motor is used to drive a hoist. When a hoist is lowered, the load accelerates the rotor above synchronous speed, and thus the motor runs as an induction generator. If the resulting speed is 1260 r/min, determine:

a. The line current.

b. The accelerating torque.

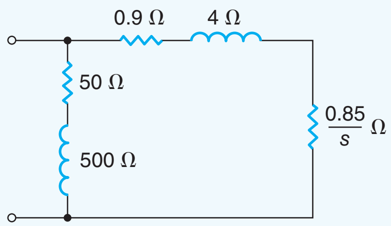

FIG. P3-11

3-12 A 3-ϕ, 480 V, three-phase induction generator delivers 375 kW when powered through a hydroturbine. The water's rate of flow and head at the turbine entrance are 4.70 m^3/s and 10 m, respectively. The open-circuit time constant of the induction generator is 4 ms, and at the sudden removal of the electrical load, the speed of the turbine reaches 2.2 pu in 1 ms.

Data from the open-circuit test of the generator are as follows:

V: 480 V, line-to-line

I: 332 A

P: 72 kW, 3-ϕ

Estimate:

a. The wire-to-water efficiency of the power station.

b. The maximum stator voltage following the removal of the electrical load.

c. The magnetizing kVAR drawn from the power distribution network to which the generator is connected.

3-13 A three-phase, 75 kW, 600 V, 60 Hz, 1160 r/min induction motor drives a pure-inertia load of 5 kg . m^2. The torque-slip characteristics of the load and the motor are as follows:

Motor torque (N · m)	200	390	455	375	250	0
Load torque (N · m)	0	60	90	120	135	150
Slip (pu)	1	0.6	0.40	0.2	0.1	0

Determine:

a. The speed of the motor at steady state.

b. The approximate time it takes the motor to reach this speed.

c. The approximate time it takes the motor to reach the speed of maximum torque.

3-14 A six-pole, three-phase, star-connected, 60 Hz, 460 V induction motor has a stator leakage impedance of $0.5 + j1.0$ ohms per phase, a rotor leakage impedance of $0.6 + j1.2$ ohms per phase, and a magnetizing impedance of $4 + j40$ ohms per phase.

a. Draw the per-phase equivalent circuit at 60 Hz and at 15 Hz.

b. Estimate the motor torque when operating at 15 Hz and a slip of 10%. Neglect the magnetizing current.

3-15 The motor in Fig. P3-15 drives a pure-inertia load and has a per-phase equivalent circuit as shown. Neglecting stator resistance and the effects of the magnetizing current, determine:

a. The energy stored in the rotating masses while the motor is running unloaded.

b. The time it takes to plug the motor to a complete stop, and the associated energy loss in the rotor windings.

c. The time it takes to reverse the speed of the motor by plugging, and the associated energy loss in the rotor windings.

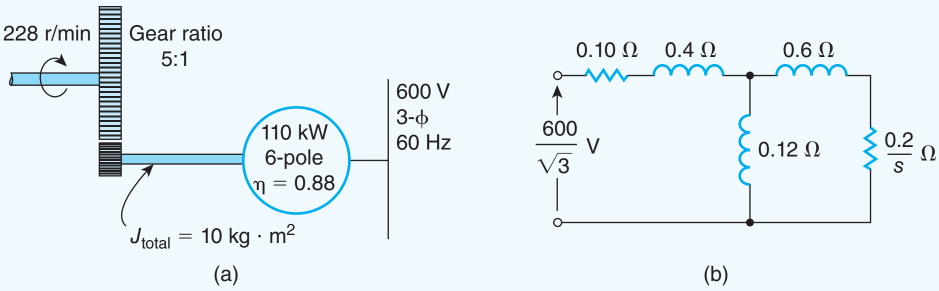

FIG. p3-15

d. The value of the rotor resistance that will furnish the minimum time for (b).

e. The number of maximum reversals per minute. Assume total motor loss to be 1.8 times the rotor copper loss.

3-16 The torque-speed characteristic of a 60 Hz, 3-ϕ squirrel-cage induction motor is controlled by a VFD. Making reasonable assumptions, prove the following:

a.

$$\frac{T_1}{T_2} = \frac{s_1}{K_1 s_2}$$

where T_1 is the torque at a slip s_1 at 60 Hz, and T_2 is the torque at slip s_2 at a frequency of 60 K_1 Hz ($K_1 < 1$).

b. For equal torques, the corresponding winding currents are equal.

3-17 The voltage output of an inverter to an inductive load is as shown in Fig. P3-17.

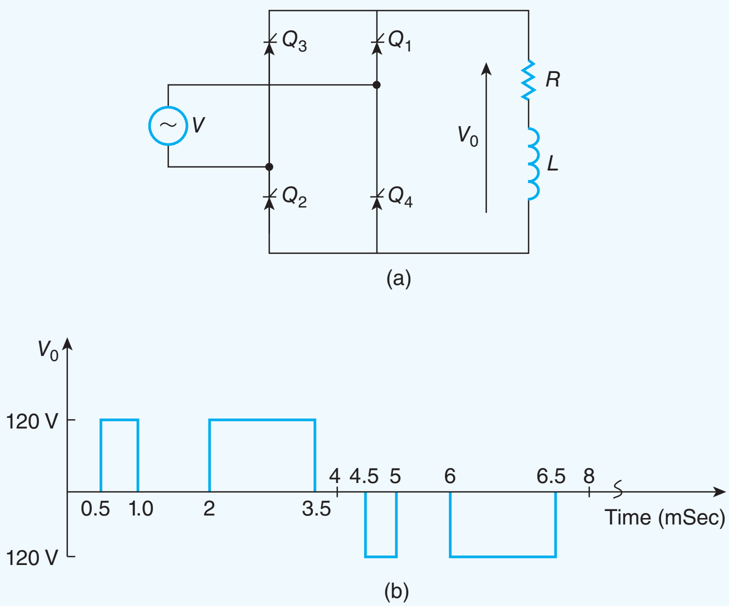

FIG. P3-17

Determine the rms value of the output voltage and its frequency.

3-18 Draw a typical characteristic of a motor and the driven fan.

a. Elaborate on these characteristics.

b. Why is the fan characteristic a second-degree curve?

(*Hint*: Pressure α V^2)

c. By increasing the air-flow rate above nominal using a VFD, what are the adverse effects?

3-19 Referring to the data in Table 3-3, compare the present cost of the energy losses of the 50 kW squirrel-cage induction motor for the two commercially available efficiencies. Assume that:

The motor operates at full-load 16 hours a day, 46 weeks a year, over a five-year period.

The energy cost is 10¢/kWh and increases by 8% per year.

The nominal interest rate is 12% and is compounded annually.

3-20 A three-phase induction motor drives a hoist. During lowering, the load accelerates the motor above synchronous speed, and thus the motor runs as a generator. The per-phase equivalent circuit of the motor under rated conditions, and its one-line diagram, are shown in Figs. P3-20(a) and (b), respectively. If the resulting speed is 1900 r/min, determine:

a. The line current.

b. The power returned to the supply.

3-21 Briefly elaborate on the following:

a. On interconnecting an asynchronous generator to a utility's network, the utility's terminal voltage must be slightly smaller than that of the asynchronous generator.

b. The impedances of the induction motor are smaller than those of the asynchronous generator.

c. How can the phase sequence of an asynchronous generator be changed?

d. Under what circumstances can a capacitor furnish the magnetizing field of an asynchronous generator, and what are its adverse effects?

e. The reactive power of the capacitor (kVAR) must be at maximum equal to the motor's reactive power.

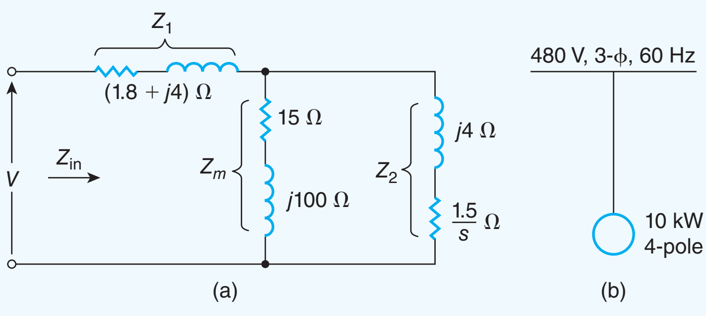

FIG. P3-20